Management strategies for an invasive species: the importance of stock externalities

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Abstract

The management of an invasive species presents some similarities with renewable resources. However, the objective function is the sum of two positive and often increasing components : environmental damages and management costs. The paper stresses the importance of stock externalities to ensure that a non zero stock is optimal. In a static approach, the paper shows that when the damage function is always increasing, the absence of stock externalities leads to a solution of eradication (zero stock) under usual assumptions. If the damage is decreasing (and negative as sometimes assumed) it is still possible that a non zero stock to be optimal. In the presence of externalities it is more likely that an interior solution be optimal, although it needs not to be the case. If the cost externalities tend to be infinitely large for low stock levels, then an eradication is ruled out. In the dynamic approach, conditions are given for an interior solution to exist. Again it is shown that the existence of externalities helps satisfy both first and second order (convexity) conditions for a solution stopping short from full eradication. An empirical illustration for *Ludwigia spp.* will be given at the conference.

JEL Classification : Q20,Q29.

Keywords: Invasive species, eradication, controlled invasion, resource economics

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Introduction

The spread of non indigenous invasive species raises important ecological and economical problems in many countries. They can be animal or plant, land or aquatic species. Biological invasions are responsible for the degradation of ecosystems and ecological services: modification of both biodiversity and habitats associated with competition among species, deterioration of water quality and changes in the flow of waters in the case of aquatic species, etc. These deteriorations of ecosystems may inflict substantial economic losses to the users of the invaded natural resources. In France, *Ludwigia spp.* is one of the most intrusive aquatic plant species, threatening the environment. World-wide, it is classified among the worst plant pests. Its dynamic power of colonization is strong in France where numerous sites exhibit conditions favorable to its growth. It holds a wide ecological magnitude and presents an important potential of geographical distribution. It prefers stagnant or weak water flow environment such as still waters and wet zones. When *Ludwigia spp.* spreads over, it is responsible for important damages on ecosystems and their functionalities. It may lower the oxygen content and alter the quality of watersheds. It also causes physical nuisances through hindrance to water flows and silting up. As a result some biotopes lose complexity and biodiversity decreases in the colonized environment. Recreational uses of natural resources such as fishing, hunting, water-based recreation, may also suffer from significant damages.

The users and managers of the affected sites are striving to develop methods to keep *Ludwigia spp.* under control. The characteristics of the species are crucial in the design of the strategy regarding management and spread risk. Some actions can have unpredicted effects on the structure of the ecosystem and its functionality. Besides, these actions tend to require costly techniques and hence large financial means. The control may consist of heavy ground works to avoid the settlement and the development of the species. Curative approaches consist of manual, mechanical or chemical techniques.

According to the managers, it seems that the complete eradication is not feasible once the colonization has started. Depending on the sites and the degree of colonization, the management objectives are either to keep the invasion at very low levels or simply to minimize the nuisances and the dispersion. However, the objective of controlling the invasion to a given physical target may not always be optimal from an economic point of view. The management level should be given by a trade off between costs and benefits.

The control of invasive species has features similar to the problems of pollution management. In both cases the spread entails damages to the society. However, unlike pollution, biological invasions face limiting factors to a species spread on a given site, due to ecological and biological conditions. In this respect they are similar to renewable resources such as fisheries stocks. But the main difference is that in the case of renewable resources extraction provides a valuable output, and the problem is to maximize the present value of the profit generated by catches. While in the case of invasive species the problem is to minimize the present value of the sum of the cost of control and of the current flow of damages generated by the stock. Eiswerth and Johnson (2002) stress that very few economic studies have been conducted on the biological invasions management. To our knowledge, (Junqueira Lopez, Michel, and Rotillon 1993) , Knowler and Barbier (2000), and Eiswerth and Johnson (2002) developed dynamic models that we can compare with those used for renewable resources.

Junqueira Lopes and al (1993), have studied a population of scrawfishes deteriorating irrigation channels. Eiswerth and Johnson (2002) have modeled an invasive plant which slows down the productivity of pasture. They derive solutions for stationary stocks of invasive species and to optimal levels of control. We note that these studies are limited to theoretical modelization and do not include fully empirical applications.

Eiswerth and Johnson (2002) developed a dynamic model of optimal control about an invasive species management. The model is close to bioeconomic models applied to fisheries and is followed by a numerical illustration. They derive optimal solutions for steady state level of invasion. The comparative statics shows that the impact of ecological, biological and technical factors on the optimal control is ambiguous and strongly depends on the species characteristics and on the colonized site. However, they do not examine the impact of economic parameters on the solution. The damage function they use depends on the invasion level, whereas the costs only are a function of the quantity withdrawn from the biomass.

The problem of controlling invasive species with a dynamic model including externalities has already been considered (Olson and Roy undated). Their contribution is to give the implications of non classical assumptions on the cost functions, the growth function and on the optimal management policy. They determine the economic and biologic conditions under which the optimal policy leads either to eradication, to inaction or to partial withdrawal. The implications in terms of optimal management are ambiguous; they can lead to complex and cyclical dynamics of control. Simulations are realized. Eiswerth and Van Kooten (2002) study the stochastic case, using probabilities from surveys of experts in the field.

The aim of this study is to identify and quantify optimal control strategies for various French sites invaded by *Ludwigia spp.* Unlike Eiswerth and Johnson, we allow for a stock externality to exist. Under reasonable circumstances, this assumption seems to be a necessary condition for the existence of a non zero-solution, hence different from eradication. A static approach is first used to identify and simulate management solutions. A dynamic model is also developed, and leads to formal steady solutions. We also examine existence and stability conditions. Although we have no general conditions on the technology and the biological characteristics which ensure that a non corner solution exists and is stable, we find that stock externalities appear to make these conditions easier to fulfill.

1 Static model : the importance of stock externalities

We consider a site invaded by species such as *Ludwigia spp.* . Their spread is responsible for nuisances or negative impacts on recreational and other uses. In order to cope with economic damage, the manager of such a site would like to control the species proliferation. He has to bear the management cost of the invaded site. We assume that the cost function depends not only on the amount of biomass retrieved but also on the invasive species stock. In the static case, the sole owner is indifferent between present and future. The manager aims at finding an efficient trade off between the current costs of control and future damages generated by the stock. The optimal management of the affected site requires to minimize the sum of control costs and damages induced by the biological invasion subject to the growth function.

Let Y be the control i.e. the amount of the invasive species extracted, and S the stock of biomass. Total and marginal economic damages are assumed to be increasing with the biomass stock. They are both zero before the invasion i.e. when the stock is nul. The damage function represented by $F(S)$ is assumed to be continuous and twice differentiable, increasing and strictly convex in S. Hence, except for specified cases, the following assumptions are made on the damage function (where indices denote partial derivatives):

$$F_S > 0\,;\; F_{SS} > 0\,;\; F(S) > 0\,;\; F(0) = F_S(0) = 0 \qquad (1)$$

Managing the invaded species incurs a cost wich in general will depend on both the 'ouput' Y and of the stock of biomass S. The latter assumption expresses the possibility of a stock externality. The assumptions on the control cost function are summarised in the following expressions:

$$C_Y(Y,S) > 0\ ;\ C_{YY}(Y,S) > 0 \tag{2}$$
$$C_S(Y,S) \leq 0\ ;\ C_{YS}(Y,S) \leq 0\ ;\ C_{SS}(Y,S) \geq 0 \tag{3}$$
$$C_{YY}C_{SS} - C_{YS}^2 > 0 \tag{4}$$

When stock externalities do not exist relations 3 are strict equalities. In the presence of externalities they are strict inequalities and we add the following realistic assumptions for extreme points:

$$C(0,S) = 0,\ S > 0\ ;\ C_Y(0,S) \geq 0,\ S > 0 \tag{5}$$
$$C(Y,0) = \infty\ ;\ C_Y(Y,0) = \infty\ ;\ C_Y(0,0) = \infty; C_S(0,0) = -\infty\ ;\ C_{YS}(0,0) = -\infty \tag{6}$$

Expression 2 implies that total and marginal costs of managing the invasive species are non decreasing in Y. Expressions 3 mean that the stock externality is weakly negative and decreasing in magnitude: total and marginal costs are non increasing in the biomass stock. The cost function is also assumed to be continuous and twice differentiable, strictly convex in Y, in S and jointly convex in S and Y, according to expression 4. Expressions 3 reflect the increasing difficulty to eradicate the species from its environment as the amount of biomass becomes scarce and, conversely, the relative easiness to remove a unit when the stock is large. We also assume in general that the cost is zero for a no control strategy and that marginal cost is a non negative number when the stock is strictly positive.

We assume a logistic growth function. Then, as the species is spreading, competition between species increases and pests and biological interactions appear. The biomass proliferates at a diminishing rate from some level and then stabilizes. The natural growth function of the species, represented by $G(S)$, is then assumed to be strictly concave with a maximum at S_M which is the maximum sustainable yield of the species. It is increasing when the stock is below S_M and decreasing otherwise. We make the usual and classical assumptions, of renewable resources management models such as fisheries, where a quadratic form is often assumed for G:

$$G_S(S) \geq 0\ ,\ 0 \leq S \leq S_M\ ;\ G_S(S) \leq 0\ ,\ S \geq S_M; G_{SS}(S) < 0\ ;\ G(0) = G(K) = 0 \tag{7}$$

When management is implemented, the natural growth of the stock is cut down by the biomass extracted, hence the following usual assumption for the net growth when the control is set at level Y:

$$dS/dt = G(S_t) - Y_t \tag{8}$$

The optimal management of the biological invasion is defined as the social planner problem. In a static approach he tends to minimize $V(S)$, the sum of damages and management costs, subject to a stationary stock biomass, i.e. to equation 8 set to zero. Then, the static optimal stock, uniquely determines the amount Y to be extracted in order to maintain the stock and keep the entailing damage under control in an efficient manner.

Replacing Y by $G(S)$ in the cost function, the problem is then reduced to find an optimum without constraint excepted for the non negativity of the stock S and the condition that the stock cannot be larger than K, as

$$\min_S V(S) = F(S) + C(G(S), S) \; ; \; S \geq 0 \; ; \; (K - S) \geq 0 \tag{9}$$

The first order necessary condition for a solution is the following:

$$V_S = F_S + C_Y \cdot G_S + C_S \geq 0; S \cdot V_S = 0 \tag{10}$$
$$-V_S \geq 0 \; ; \; (K - S) \cdot V_S = 0 \tag{11}$$

if $V_S > 0$ for all S, then S^* is zero and the optimal solution is eradication,

if $V_S < 0$ for all S, then $K - S^* = 0$ and the optimal solution is *laisser faire*,

if $V_S = 0$ for some S and $V(S)$ is strictly convex, then $0 < S^* < K$ and the optimal solution is controlled invasion.

The sufficient condition for an interior solution to be a true minimum is:

$$V_{SS} = F_{SS} + C_{YY} \cdot G_S^2 + C_{SS} + 2C_{YS} \cdot G_S + C_Y \cdot G_{SS} > 0 \tag{12}$$

Assumptions 2 to 4 do not ensure that this condition is always met. The value function $V(S)$ could in principle be increasing for the whole range of stock levels. In order to minimize the total loss, the condition 10 requires the stock to be zero if the value function $V(S)$ is strictly increasing in S. There is no interior solution. Indeed, it turns out that it is the most likely case when the damage function is strictly increasing and when the stock externality is either zero or small in comparison with the marginal damage. As a result, the planner should either prevent the species from proliferation at the outset or completely eradicate the biomass on the site subject to invasion. The absence of an externality does not guarantee that the function $V(S)$ is strictly increasing because when $S > S_M$, the natural production function is decreasing ($G_S < 0$). Nevertheless, even in that case, it turns out that a weaker condition justifies eradication under our realistic assumptions. Let's examine different cases leading to several types of control strategies.

1.1 No stock externality

Without stock externalities, the necessary condition for a minimum is :

$$V_S = F_S + C_Y \cdot G_S \geq 0\,;\; S \cdot V_S = 0 \tag{13}$$

For $S < S_M$, G_S is strictly positive and the value function is increasing. Then, an optimal solution is not possible in this interval. When $S > S_M$, V_S is likely to have a zero value. However, it is not a minimum even if we can not conclude from the simplified second order condition:

$$V_{SS} = F_{SS} + C_{YY} \cdot G_S^2 + C_Y \cdot G_{SS} > 0 \tag{14}$$

This condition is not guaranteed because in theory, the concavity of G can offset the other two positive terms and the value function $V(S)$ can either keep on increasing after a turning point or be decreasing after a maximum. This will be clearly the case if marginal damages and marginal costs are constant.

Nevertheless, we can check that, in the absence of stock externalities, the loss function V has a global minimum at a zero stock level if damage function is non decreasing as assumed :

$$V(S) = F(S) + C(G(S), S) \geq V(0) = F(0) + C(0,0) = 0 \tag{15}$$

Result 1: *In a static approach, whenever stock externalities are absent and the damage function is non decreasing, the optimal solution is to drive the stock to zero i.e. to eradicate completely the invasive species from the environment.*

If for low levels of the invasion damages are decreasing over some range and then increasing, as the weeds may provide shelter to fish for spawning, we may still have an interior solution. Expressions 13 can then be zero for some non zero S and be a minimum of $V(S)$ as the second order condition 14 is negative. Such a case, which violates the more relevant assumptions stated in 1, will occur for some species which first provide benefits so that damages are negative and decreasing for small values of the stock biomass, then increasing after a minimum and then positive. As a result, a non zero stock can be optimal and the optimal policy will not be eradication.

Result 2: *In a static approach, whenever stock externalities are absent, it is necessary for the optimal solution to occur at a (strictly) positive stock level that the damage function be decreasing over some range.*

With no stock externalities and an increasing damage function the first order conditions do no provide the global minimum but a maximum, or a turning point. Eiswerth and Johnson (2004) used cost functions which are independent of the colonization size, hence they can not ensure that their solution is a true minimum even if the model is dynamic.

1.2 Stock externalities

When it is hardly feasible or impossible to eliminate the last individuals, eradication costs become prohibitive as the stock becomes very low. The optimal management may consist of maintaining the stock at the positive level derived from expression 10, which means that the optimal stock is such that the marginal savings on costs due to an extra unit of stock should offset the corresponding increase in damage.

$$-\frac{dC}{dS} = -[C_Y \cdot G_S + C_S] = F_S \tag{16}$$

This condition can also be written as $G_S = -[F_S + C_S]/C_Y$ which requires the marginal productivity of the stock (the slope of the growth function) to be the opposite of ratio of the marginal damage net of the stock externality to the marginal cost of extraction. Depending on the relative importance of externalities and marginal damages, G_S is positive or negative, hence the optimal solution can then be on the left or on the right of S_M.

This condition leads to a non zero solution for S, as long as the second order condition 11 is also met for all positive S. We cannot conclude about the convexity of $V(S)$ under the weak assumptions made in 1 to 4. Given these assumptions on cost and damage functions, the first three terms are positive. The fourth is negative if $S < S_M$ and positive otherwise. The last term is always negative. The sign of V_{SS} is thus ambiguous. Consequently, the conditions to get a non zero optimal solution S^* are fairly restrictive. We may add that strongly increasing damage function (large F_{SS}) and rapidly decreasing externalities in absolute value (large C_{SS}) make an interior solution more likely. The nature of the solution, eradication, controlled invasion, or *laisser faire* seems very sensitive to parameters and to the functional forms for costs, damages and growth. Regulation and action levels are different and strongly depend on the characteristics of the colonized sites. As a matter of illustration in figure 1 and 2 we provide typical forms of the $V(S)$ curve under different parameters of the following functionnal forms:

$$C(Y, S) = (\alpha Y + 0.5\beta Y^2)/(S/K)^u \quad (17)$$
$$F(S) = \delta + \varphi(S - \tau)^2 \quad (18)$$
$$G(S) = \gamma S(1 - S/K) \quad (19)$$

Figure 1 corresponds to the case of no externalities ($u = 0$). A monotonic damage function ($\tau = 0$) gives curves $V1(S)$ and $V12(S)$ which are two cases for eradiction. A damage function, decreasing for low stock levels and increasing after (curve $V11(S)$), may lead to an interior solution. Figure 2 synthetises the main cases with a strictly monotonic damage function. Curves $V1(S)$and $V11(S)$ are just the previous ones with no externalities. Curves $V2(S)$ and $V4(S)$ correspond to stock externalities. $V2(S)$ illustrates the corner solution of *laisser faire* $S^* = K$ (damages are small compared to costs, and ($dC/dS + F_S$ is always negative). $V4(S)$ is a combination of strong externalities and large and increasing damages.

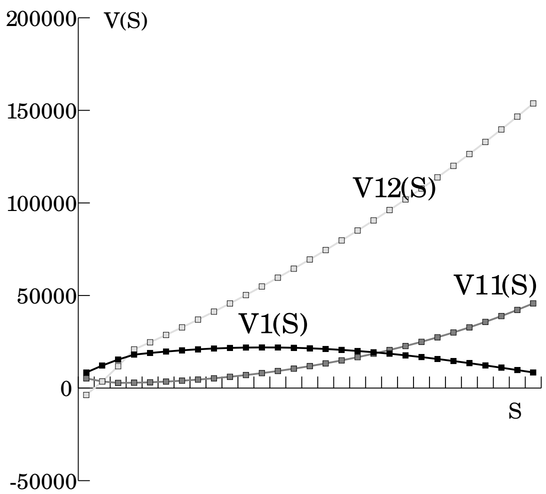

Figure 1: No stock externalities: two corner solutions ($S* = 0$: Eradication) and a case of positive solution (F_S negative for small S)

1.3 A few empirical results

To complete the empirical application to *Ludwigia spp.* we need to estimate the cost, damages and growth functions. To that purpose, managers who are in charge of controlling the spread were surveyed to collect data, identify strategies and assess the benefit cost efficiency of the actions. So far, data

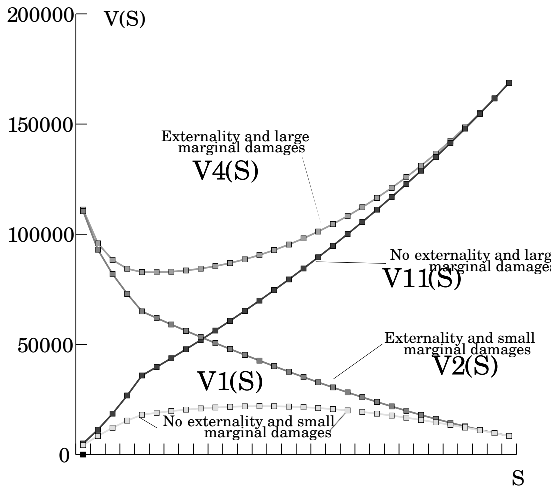

Figure 2: Four Strategies for selected cases: two with and two w/o externalities

are too scarce to estimate a cost function. Nevertheless we have calibrated a cost function for both manual (51 to 64 Euro/tonne) and mechanical (1100 to 1300 Euro/tonne) techniques (figure 3). This heterogeneity is due to differences between sites characteristics and degree of invasion.

1.4 Comparative statics

We examine two schocks on the optimal solution: an increase in the cost $C(Y,S,\alpha)$ due to an input price change $d\alpha$ and a shift $d\delta$ in the damage function $F(S,\delta)$. These two schocks relate to economic variables and are relevant to an interior solution, where V_{SS} is strictly positive. We note that Eisworth and Johnson (2004) studied shocks on the technology only. Total differentiation of 10 gives :

$$V_S dS + F_{S\varphi} d\varphi + G_S C_{Yw} dw + C_{Sw} dw = 0 \tag{20}$$

Hence the impacts of the shocks on the optimal stock:

$$\partial S^*/\partial w = -\frac{C_{Yw} \cdot G_S + C_{Sw}}{V_{SS}} \geq 0 \tag{21}$$

$$\partial S^*/\partial \varphi = -\frac{F_{S\varphi}}{V_{SS}} \leq 0 \tag{22}$$

The partial effect of an input price on optimal stock is then positive under usual assumption $C_{Yw} \geq 0$ et $C_{Sw} = C_{wS} \leq 0$, with no doubt

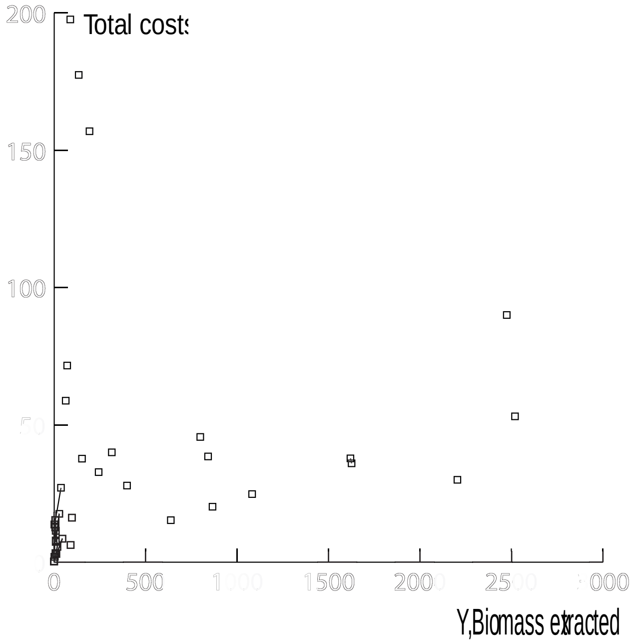

Figure 3: Some empirical evidence on total costs of control

if $S^* > S_M$ or when the the stock externality effect on input demand is strong. This is consistent with a negative effect on the control effort Y. When the valuation of damages increase, the optimal stocks should also be cut down. Our comparative statics of exogenous economic shocks are therefore consistent with economic intuition.

2 Solutions for the dynamic model

In the dynamic approach the manager has a preference for the present and faces the trade off between current and future value of damages and costs. A typical formulation of the problem in renewable resource is to minimize the discounted value of the losses subject to the dynamics of the growth function. Our model is similar to that of Eiswerth and Johnson (2002) except that we allow for stock externalities, which seems to be relevant to our case study. The problem is written as

$$\max_{Y_t} - \int_0^\infty e^{-rt}(F(S_t) + C(Y_t, S_t))dt \tag{23}$$

subject to the non negativity of the control variable Y_t, to an initial value of the stock S_0 and to the growth function 8. The general conditions 1 to 6 on the damage and cost functions are also assumed. To solve this classical optimal control problem with an infinite time horizon we construct the current value Hamiltonian (where time subscript is deleted).

$$\widetilde{H} = -F(S) - C(Y, S) + \mu(G(S) - Y) \tag{24}$$

Where μ is the current value of the costate variable, i.e. the shadow value of the stock which should be negative on the optimal trajectory. μ and the present value costate variable, called λ, are related through $\lambda = \mu e^{-rt}$. An optimal trajectory must satisfy the following conditions for a maximum:

$$\frac{\partial \widetilde{H}}{\partial Y} = -C_Y(Y, S) - \mu \leq 0, Y \geq 0 \tag{25}$$

$$Y \cdot (C_Y(Y, S) + \mu) = 0 \tag{26}$$

$$\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial S} = -\mu G_S + F_S + C_S \tag{27}$$

$$\frac{\partial \widetilde{H}}{\partial \mu} = \dot{S} = G(S) - Y \tag{28}$$

$$\lim_{t \to \infty} \mu \cdot e^{-rt} \cdot S = 0 \tag{29}$$

The necessary condition 25 says that the marginal cost of effort should be equal to the (negative of the) shadow cost of the biomass stock for the optimal control to be non zero .

$$(a) C_Y() > -\mu \Rightarrow Y = 0 \tag{30}$$

$$(b) C_Y() = -\mu \Rightarrow Y > 0 \tag{31}$$

If the cost function is strictly convex in Y, the optimal value of Y will be unique and between zero and the maximum admissible for Y (no bang-bang solution). Under strictly increasing marginal costs, expression 31 can be inverted to give the optimal value of Y as a function of μ and S.

$$C_Y(Y, S) = -\mu \Leftrightarrow Y(\mu, S) > 0 \tag{32}$$

This function is the (restricted) supply function of control effort Y in terms of the shadow price μ and the stock. From differentiation of 31 and the assumption of diminishing returns, Y appears to be increasing in $-\mu$ and increasing in S.

$$\frac{\partial Y(\mu, S)}{\partial S} = -\frac{C_{YS}}{C_{YY}} > 0 \tag{33}$$

$$\frac{\partial Y(\mu, S)}{\partial \mu} = -\frac{1}{C_{YY}} < 0 \tag{34}$$

Another necessary condition for a unique (interior) solution to be a maximum of problem 21 is that the Hamiltonian is strictly concave with respect to the state variable S along the optimal trajectory ($\mu \neq 0; Y = G(S)$) , i.e.

$$\widetilde{H}_{SS} = -(F_{SS} + C_{YY} \cdot G_S^2 + C_{SS} + 2C_{YS} \cdot G_S + C_Y \cdot G_{SS}) < 0 \qquad (35)$$

A stationary steady state solution should verify this condition which is formaly the same as the second order condition of the static case ($V_{SS} > 0$). Again, our general assumptions 1 to 7 on the functions do not ensure it is fulfilled. It will be more likely satisfied if the damage and cost functions are strongly convex in S and Y and if the solution occurs when G_S is negative. Strong stock externalities help satisfy this condition. We now study the existence and stability of a stationary solution under the general assumptions.

A solution should verify both differential equations 27 and 28, with Y defined in 32 by the necessary condition 26. A stationary interior solution, if it exists, corresponds to $\dot{S} = 0$ and $\dot{\mu} = 0$, and should satisfy:

$$\dot{\mu} = (r - G_S)\mu + C_S(Y(\mu, S), S) + F_S = 0 \qquad (36)$$

$$\dot{S} = G(S) - Y(\mu, S) = 0 \qquad (37)$$

If a non zero stationary solution (μ^*, Y^*, S^*) exists, it will therefore be such that

$$\mu = \frac{C_S + F_S}{(G_S - r)} = -C_Y \qquad (38)$$

Expressions 36 and 37 define two relations between μ and S in the (μ, S) space. We call them $\mu^\mu(S)$ and $\mu^S(S)$ respectively to trace their origin. We first study the corresponding curves on the basis of slopes.

$$\frac{d\mu^S}{dS} = -\dot{S}_S / \dot{S}_\mu = -(C_{YY}G_S + C_{YS}) \qquad (39)$$

$$\frac{d\mu^\mu}{dS} = -\dot{\mu}_S / \dot{\mu}_\mu = \frac{-C_{YY}C_YG_{SS} - F_{SS}C_{YY} - (C_{YY}C_{SS} - C_{YS}^2)}{(r - G_S)C_{YY} - C_{YS}} \qquad (40)$$

The slopes of both curves are not clear cut under general conditions. We have to consider subcases related to externalities and extreme points when $S = 0$ and $S = K$, in order to identify likely solutions.

2.1 No stock externality

Without stock externality 39 simplifies and the slope of μ^S is the opposite of that of the growth function of which it has a symmetrical shape. For

$S = 0$, Y also has to be zero, hence we must by 30 also have $\mu > -C_Y(0)$ which is a finite negative number, say $-\alpha$, in regards to the particular case described in functions 17. Because we have no externality, μ^S takes the same value for $S = K$. Hence the shape of the μ^S curve in figure 4.

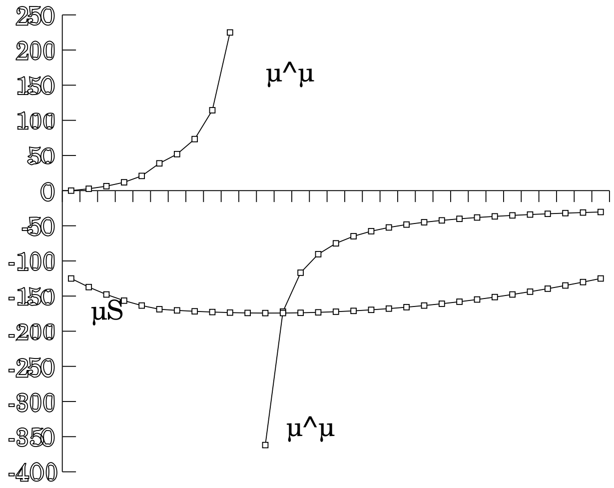

Figure 4: Corner solution ($S^* = 0$: Eradication, no stock externalities, $u = 0$)

The slope of the μ^μ function also simplifies to $(-C_Y G_{SS} - F_{SS})/(r - G_S)$. The sign of the numerator is ambiguous. The denominator is negative (and positive otherwise) when S is below a critical level where the μ^μ will have an asymptote. This critical level $\tilde{S}$ is defined by:

$$G_S(\tilde{S}) - r = 0 \tag{41}$$

Consider first the domain where $0 < S < \tilde{S}$. For $S = 0$, μ^μ is positive (or zero if $F_S(0) = 0$). When $S \to \tilde{S^-}$, $\mu^\mu \to +\infty$. In this domain we have a possible solution at $S^* = 0$ since $\mu^\mu(0) \geq 0 > -C_Y(0) = -\alpha$. The other segment of the μ^μ-curve will start from the asymptote at $\tilde{S}$ where $\mu^\mu \to -\infty$ when $S \to \tilde{S^+}$. By 38, μ^μ will have a finite negative value for $S = K$, namely $F_S(K)/(G_S(K) - r)$ i.e., $-2\varphi K/(\gamma + r)$ in our example. For this curve to cross the μ^S-curve, this will have to be greater than $-C_Y(0) = -\alpha$. If this is the case, the two curves intersect in this domain, say at $S^{'}$. However for this to be a true maximum, the concavity of the simplified Hamiltonian in 42 should be verified.

$$\widetilde{H}_{SS} = -(F_{SS} + C_{YY} \cdot G_S^2 + C_Y \cdot G_{SS}) < 0 \tag{42}$$

This condition is in general ambiguous and depends on functions and parameters. In the simulation which has generated figure 4, this virtual solution turned out to violate the concavity condition and was not an optimum.

When $S = K$, the necessary condition 30 is also satisfied, it is therefore a solution candidate. We seem to have at first glance three possible solutions. Can we still have a strong conclusion as in the static case which led to eradication? Yes, since the steady state current value (and present value as well) function at $S^{'} > 0$ is necessarily a larger negative number than at $S = 0$ and $Y = 0$ since $-F(S^{'}) - C(Y^{'}) < -F(0) - C(0)$, hence any intersection between the μ-curves at an interior stock level does not give an optimum. K is not an acceptable solution either since $-F(K)-C(0) < -F(0)-C(0)$. Therefore the only optimal solution in the dynamic approach without externality and with an increasing damage function is also $S^* = 0$.

This no externality solution is consistent with the static case and implies eradication. Again, if we had a decreasing damage function over some low stock level, the critical value of $\mu^{\mu}(0)$ might be negative so that the two μ^{μ}-curve start from a negative intercept and therefore might intersect (but $F_{SS} < 0$ makes the concavity condition 42 harder to fullfil).

2.2 Negative stock externality

The shape of the μ-curves will be different when stock externalities prevail. The slope of the μ^S-curve in 39 does not have an obvious sign throughout, unless reasonable assumptions are made on the externality. However assumptions made in 6, ensure that as S approaches zero $\mu = -C_Y(0,0) \rightarrow -\infty$ and the slope of the μ^S-curve is positive and large ($C_{YS}(0,0) \rightarrow -\infty$). When $S = K$, $\mu = -C_Y(0,K)$ which is a finite negative number. When $S > S_M$, the slope is positive. When $S < S_M$, this will also be positive if externalities are large so as to offset the first term. Hence the μ^S-curve in figure 5.

The slope of the μ^{μ}-curve was given in 40. The sign of the numerator is ambiguous. The denominator is clearly positive when $S > \widetilde{S}$, but the slopes keeps its ambiguity. By 38 there is an asymptote at $S > \widetilde{S}$. If the externality dominates the marginal damage i.e. $|C_S| > |F_S|$, $\mu^{\mu} \rightarrow +\infty$ when $S \rightarrow \widetilde{S}^{+}$ and $\mu^{\mu} \rightarrow -\infty$ when $S \rightarrow \widetilde{S}^{-}$. It is the converse in the opposite case. When $S = 0$, $\mu^{\mu} \rightarrow (C_S(0,0) + F_S(0))/(G_S(0) - r) = -\infty$, under assumptions [1] 1 and 6 corresponding to our functions used for numerical simulations and the graphic illustration in figure 5.

With our functionnal forms in 17 to 19, we have obtained a non zero solution corresponding to both strong externalities and rapidly increasing

[1]Other forms could sensibly imply that this limit is zero. This does not preclude our solution to satisfy existence and stability)

damages. These are conditions which favor the prevalence of controlled invasion strategies. The non zero solution produced in figure 5 has proved to satisfy both the condition of concavity of the Hamiltonian and also the local stability condition for a saddle point described in the appendix. It is easy to produce a subcase with externality where eradication is recommanded as it was the case in figure 4. We can also easily produce a case where costs dominate damages and *laisser faire* at $S = K$ is the most sensible policy.

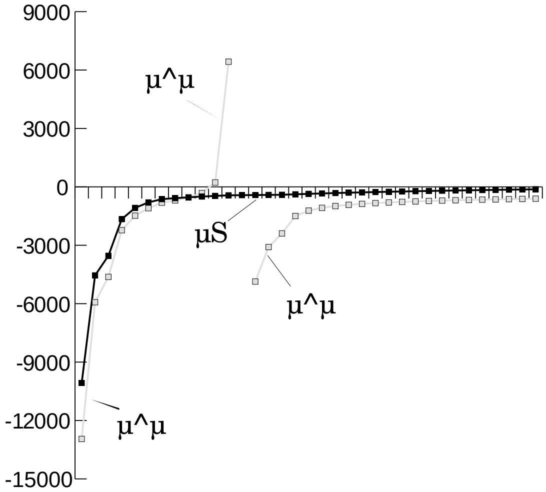

Figure 5: Interior solution (Stock externalities, u=1.2)

3 Conclusion

The modelling of invasive species management seems to have made little use of externalities. The first order conditions for discovering the optimal management strategy of an invasive species are to be used with caution. In the absence of such externalities, non zero optimal solutions in steady state are unlikely to fulfil second order conditions and to be stable. We have established the conditions for different strategies in the static approach and sketched the likely structure of the solutions in the dynamic case. An empirical application to *Ludwigia spp.* is in progress.

From an empirical view point, the analysis shows that eradication will always be the best policy if no stock externality exists and if damages are non decreasing. It does make sense, when it is easy to catch and destroy the ultimate individuals of an invasive species, to opt for preventive policy which consists of eradication. This case seems to be fairly rare. If however, the costs of catching these last individuals tend to sharply raise, i.e. in the presence of severe stock externalities, the best policy is likely to be a population control at a positive level and to regularly destroy a certain amount of the biomass to keep the population stable. If damages are small and weakly increasing and if externalities are significant, a *laisser faire* policy is more likely to prevail. The existence of stock externalities are in reality nearly necessary conditions to encounter a non corner solution.

A Stability of the steady state interior solution

The optimal control problem is autonomous with a positive discount rate. The solution can only be locally stable along a defined time path and be, at best, a saddle point. Assuming a steady state exists, the local stability is studied by linearising the differential equations system around equilibrium. Taking a first order linear approximation around the steady state, we get:

$$\begin{pmatrix} \dot{S} \\ \dot{\mu} \end{pmatrix} = \begin{pmatrix} \dot{S}_S(*) & \dot{S}_\mu(*) \\ \dot{\mu}_S(*) & \dot{\mu}_\mu(*) \end{pmatrix} \cdot \begin{pmatrix} (S - S*) \\ (\mu - \mu*) \end{pmatrix} + \begin{pmatrix} \epsilon(S*, \mu*) \\ \epsilon'(S*, \mu*) \end{pmatrix}$$

where $\epsilon(S*, \mu*) \rightarrow 0$ and $\epsilon'(S*, \mu*) \rightarrow 0$. If the characteristic roots of the matrix are real and of opposite sign, then the equilibrium is a saddle point. The correponding determinant has to be negative, i.e.:

$$\begin{aligned} G_S(r - G_S) - (C_Y \cdot G_{SS} + F_{SS})/C_{YY} + (C_{YS}(r - G_S) \\ + C_{YS} - C_{YY} \cdot C_{SS})/C_{YY} < 0 \end{aligned} \quad (43)$$

A sufficient condition is that, at steady state, the current Hamiltonian is concave and also that either the stock is larger than S_M, or the discount rate is low. The concavity of the Hamiltonian was seen to be uncertain. However, as for existence of an interior solution, the presence of stock externalities strengthens the negativity of the determinant.

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