Extension of the Effective Medium Approximation for Determination of the Permeability Tensor of Unsaturated Polycrystalline Ferrites

David Bariou, Patrick Quéffélec, *Member, IEEE*, Philippe Gelin, and Marcel Le Floc'h

***Abstract*—This paper presents a physical model of the properties of polycrystalline ferrites below magnetic saturation, a common condition in many applications of ferrites in microwave devices. The properties are mainly characterized through the elements of the effective permeability tensor as functions of magnetization state, anisotropy field, and frequency. Partially magnetized states are characterized by a suitable distribution of magnetic domains over orientations. The magnetic domain shapes studied were cylinders and spheres. Homogeneity of the medium is obtained in the effective medium approximation, which allows us to treat heterogeneous magnetic materials as a function of the volume fraction of nonmagnetic matter present in the material. The model gives all the components of the permeability tensor in a single calculation phase. The paper presents results for different partially magnetized states at remanence (with no external field applied) and compares them with empirical formulations of permeability tensor components, in their domain of validity.**

***Index Terms*—Anisotropic media, ferrimagnetic materials, ferrites, permeability.**

I. Introduction

PARTIALLY magnetized ferrites are used in devices such as circulators, latching phase shifters, and tunable yttrium iron garnet filters. As computer-aided design plays an important role in the study of microwave devices, it is necessary to have a characterization of the ferrite material that describes the partially magnetized state.

For technological reasons, ferrites are often used in polycrystalline form, i.e., the direction of the crystal symmetry axis varies randomly from point to point. In a partially magnetized state, the magnetic domains may be oriented along the various easy axes of the grains that constitute the heterogeneous medium. It is quite difficult to analyze the microwave response of partially magnetized ferrites, because the magnetic domains have neither a well-defined shape nor a well-defined magnetization direction. Furthermore, the response characteristics may change as a function of the dc magnetic field. Moreover, the different domains interact dynamically. Interaction causes a behavior such as Polder–Smit resonances [1], which are characterized by high losses over a range of frequencies.

Over the years, various approaches have been tried to measure and characterize the permeability tensor. Green and Sandy [2], following experimental work, have developed empirical formulas valid in the low-loss region ($\omega \gg \omega_m = \gamma\mu_o M_s$, where γ is the gyromagnetic ratio, μ_o is the permeability of free space, and M_s is the saturation magnetization) to characterize the permeability tensor components as functions of parameters such as M_s, reduced magnetization $m = M/M_s$, and frequency. Existing models for unsaturated ferrites imply simplifications either in the domain distribution—for example, choosing only "up" and "down" domains [3]–[5]—or considering the domains as independent [6] (interactions are neglected). Recently, our research group has developed a model taking interactions between adjacent domains into account and giving all the permeability tensor components for different magnetization states [7]. However, this approach is not easy to extend toward the description of heterogeneous magnetic material, which is our final goal. Because of the complexity of the real medium, exact calculations on complex magnetic-domain geometry become quickly intractable. Consequently, a better description of internal medium requires useful method of approximation for calculating permeability tensor.

A usual scheme for estimating the effective properties of a heterogeneous medium is known as the effective-medium approximation (EMA) [4], [8], [9]. EMA is often used to study the effects of the dilution of an active material embedded in a host matrix. Composite materials made of ferrimagnetic or ferromagnetic grains embedded in a nonmagnetic host could also be studied using the same scheme. Those materials present the following advantages over usual bulks ferrites: they are easier to machine, do not require high temperatures for sintering, and high saturation magnetization can be reached with practical applications in microwave, under the condition of macroscopic insulation in the case of ferromagnetic grains. Here we will apply the EMA to the description of an assembly of subregions (magnetic domains) with differently oriented permeability tensors, and so do not consider the presence of nonmagnetic inclusions. The effective properties of a heterogeneous material depend on the internal geometry, i.e., the domains' distribution over space and the domain shape, as well as the intrinsic properties of the components such as M_s or anisotropy field H_a. For this reason, we will discuss within the EMA an idealized domain model, but not limited to up and down domains as the previous ones [10]. We provide results for domains that are spherical and cylindrical in shape and perform the calculations for demagnetized and partially magnetized media. Magnetization of the medium is ob-

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D. Bariou, P. Quéffélec, and M. Le Floc'h are with the Laboratory for Electronics and Systems of Telecommunication, Université de Bretagne Occidentale, U.F.R. Sciences, 29285 Brest cedex, France.

P. Gelin is with the Laboratory for Electronics and Systems of Telecommunication, ENST de Bretagne, 29285 Brest cedex, France.

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tained by a suitable distribution of the magnetic domain orientation, representative of various remanent states on the hysteresis loop.

II. Theory

A. Description of the Polycrystalline Medium

Partially magnetized ferrites can be characterized by an effective permeability tensor depending on local properties. The local susceptibility can be determined from the magnetic equation of motion, knowing the direction of the local saturation magnetization. From the Landau–Lifshitz–Gilbert equation, the permeability tensor for one domain is of the following form:

$$\hat{\mu} = \begin{bmatrix} \mu & +j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

with

$$\mu = 1 + \frac{\omega_m \cdot (\omega_o + j\alpha\omega)}{(\omega_o + j\alpha\omega)^2 - \omega^2} = \mu' - j\mu'' \tag{2}$$

and

$$\kappa = \frac{\omega_m \cdot \omega}{(\omega_o + j\alpha\omega)^2 - \omega^2} = \kappa' - j\kappa'' \tag{3}$$

where α is the damping factor, $\omega_o = \gamma H_o$, and H_o is the effective dc magnetic field strength in the direction of the local dc magnetization (crystalline anisotropy).

The microwave behavior of the bulk ferrite can generally be characterized by an effective (sometimes also called global or macroscopic) permeability tensor [10]. For a material magnetized in the z direction, the linear permeability takes the following form:

$$\hat{\mu}_e = \begin{bmatrix} \mu_e & +j\kappa_e & 0 \\ -j\kappa_e & \mu_e & 0 \\ 0 & 0 & \mu_{e_z} \end{bmatrix}. \tag{4}$$

The components μ_e, κ_e, and μ_{e_z} depend on frequency as well as on the location on the hysteresis loop. One of the main difficulties is to link the permeability tensor to different magnetization states from the knowledge of local properties. To predict the value of the macroscopic magnetization, the distribution of domain magnetization directions at each point of the hysteresis loop must be established. Magnetization is essentially achieved by a change in the direction of domain magnetization through mechanisms like rotation of the spins or domain wall displacement under dc magnetic field. To carry out calculations, it was necessary to assume a simple domain distribution. We consider a material that has randomly oriented axes of magnetic anisotropy. In the demagnetized state, the material has its domain magnetization isotropically distributed. In the case of uniaxial magnetic anisotropy, the distribution function f of the magnetic domains over space is given by [11]

$$\begin{aligned} f(\theta) &= \sin\theta \quad \text{if } \theta \in [0, \theta 1] \\ &= \frac{1}{2}\sin\theta \quad \text{if } \theta \in [\theta 1, \pi - \theta 1] \\ &= 0 \quad \text{if } \theta \in [\pi - \theta 1, \pi] \end{aligned} \tag{5}$$

where θ is the deviation angle of magnetic domains from Oz axis, which is the direction of static magnetization (Fig. 1). The reduced magnetization of the material is linked to θ_1 by the formula

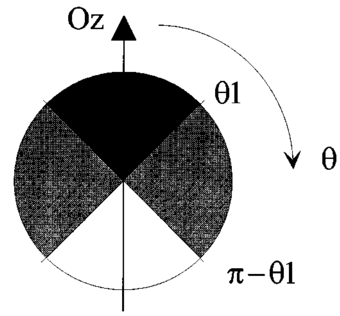

Fig. 1. Distribution of magnetic domains over directions given by (5), representative of isotropic polycrystalline ferrite. Black: $f(\theta) = \sin\theta$; gray: $f(\theta) = (\sin\theta)/2$; white: $f(\theta) = 0$.

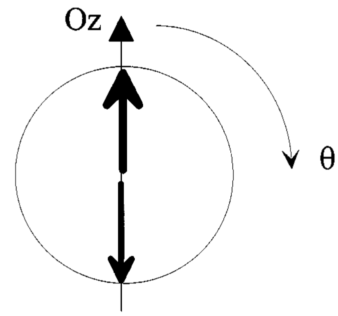

Fig. 2. Distribution of magnetic domains over directions given by (7), representative of uniaxial ferrimagnet. Up arrow: $f = (1+m)/2$; down arrow: $f = (1-m)/2$.

$$m = \frac{M}{M_s} = \langle \cos\theta \rangle = \int_0^\pi f(\theta)\cos\theta d\theta = \frac{1}{2}\sin^2\theta 1. \tag{6}$$

The distribution function $f(\theta)$ in (5) is normalized: $\int_0^\pi f(\theta)d\theta = 1$ and representative of different remanent states of isotropic polycrystalline ferrites. Then, it applies only for $0 < m < 0.5$. For comparison, Bouchaud and Zerah [4], in their model of a uniaxial ferrimagnet, used the following function (Fig. 2):

$$f(\theta) = \frac{1+m}{2}\delta(\theta) + \frac{1-m}{2}\delta(\theta - \pi). \tag{7}$$

In the following, we will study cylindrical and spherical shapes. Once geometry, distribution [(5)], and intrinsic properties (M_s, H_a, α) are fixed, we have to calculate the effective magnetic properties of such heterogeneous media. Many physical phenomena in heterogeneous media can be described by an effective medium theory [8], [9].

B. The Effective Medium Approximation (EMA)

The EMA links the local characteristics to the effective ones, taking into account interactions in a suitable way. Schematically, this theory involves embedding heterogeneities, i.e., magnetic domains in our case, in a self-consistently determined background medium. We review the principal results of EMA in this section.

Assume that a heterogeneous medium is characterized in each magnetic domains by a tensor $\hat{\mu}(\mathbf{r})$, which takes the form of (1). Measurable properties are characterized by

$$\langle b \rangle = \hat{\mu}_e \cdot \langle h \rangle \tag{8}$$

where brackets indicate spatial averaging over orientations. Equation (8) defines the effective permeability tensor $\hat{\mu}_e$ of the medium, which is independent of position and takes the form of (4). The magnetic field inside the medium is given by [8]

$$h(r) = h_o + \int \hat{\Omega}(r, r') \cdot [\delta\hat{\mu}(r') \cdot h(r')] \cdot d^3r \qquad (9)$$

where h_o is the externally applied field, $\delta\hat{\mu}(r) = \hat{\mu}(r) - \hat{\mu}_e$, and the (β, γ) components of $\hat{\Omega}(r, r')$ can be written as

$$\Omega_{\beta\gamma} = \frac{\partial}{\partial r'_\beta}\frac{\partial}{\partial r_\gamma} G(r - r'). \qquad (10)$$

The Green function G satisfies

$$\vec{\nabla} \cdot [\hat{\mu}_e \vec{\nabla} G(r - r')] = -\,\delta(r - r') \qquad (11)$$

$$G(r - r') = 0 \text{ on } S \qquad (12)$$

where S is the surface of the sample. Equation (9), giving the magnetic field inside the medium, is approximately decoupled by considering one domain embedded in a homogeneous, possibly anisotropic, effective medium. The EMA is then written as follows, writing $\hat{\mu}(r) = \hat{\mu}_i$ when $\mathbf{r}$ is over the ith domain:

$$\left\langle \left[\hat{1} - \delta\hat{\mu}_i \cdot \hat{\Gamma}_i\right]^{-1} \cdot \delta\hat{\mu}_i \right\rangle = \hat{0} \qquad (13)$$

with $\delta\hat{\mu}_i = \hat{\mu}_i - \hat{\mu}_e$. The (β, γ) components of the demagnetizing tensor $\hat{\Gamma}_i$ are defined by

$$\Gamma_i^{\beta\gamma} = -\int_{V'} d^3x' \cdot \Omega_{\beta\gamma} \qquad (14)$$

where V' refers to the volume of the ith domain. As shown in Appendix I, it is possible to link $\hat{\Gamma}_i$ to the particle shape tensor $\hat{N}$

$$\Gamma_i^{\beta\gamma} = -\frac{1}{\sqrt{\mu_{e\beta} \cdot \mu_{e\gamma}}} \cdot N_i^{\beta\gamma} \qquad (15)$$

where $N_i^{\beta\gamma}$ is the shape particle tensor component in the rescaled coordinate system $[O, X, Y, Z]$ given by

$$X = \frac{x}{\sqrt{\mu_e}} \quad Y = \frac{y}{\sqrt{\mu_e}} \quad Z = \frac{z}{\sqrt{\mu_{e_z}}}. \qquad (16)$$

Equation (13) leads to three nonlinear equations that are not independent. An optimization procedure is required to solve the problem. For the case in which the domains are infinite cylinders oriented along the z axis, considered in (7) and [4], the rescaling does not change the particle shape and leads to simple analytical formulations. In that case, the component corresponding to the z-direction always equals unity, and so a distribution over orientation of anisotropy axes need not be taken into account.

III. Results

The results we present are obtained by solving (13), using (5) for the averaging. Physical solutions are obtained by imposing continuity on the whole spectrum, and the appropriate loss terms to be positive [see (23)]. We have studied two domain geometries: cylinder and sphere, for different magnetization states between $m = 0$ and $m = 0.5$. All the results have been obtained with the following parameters:

$$f_m = \frac{\omega_m}{2\pi} = 10 \text{ GHz}, \quad f_o = \frac{\omega_o}{2\pi} = 1 \text{ GHz}, \quad \alpha = 0.1.$$

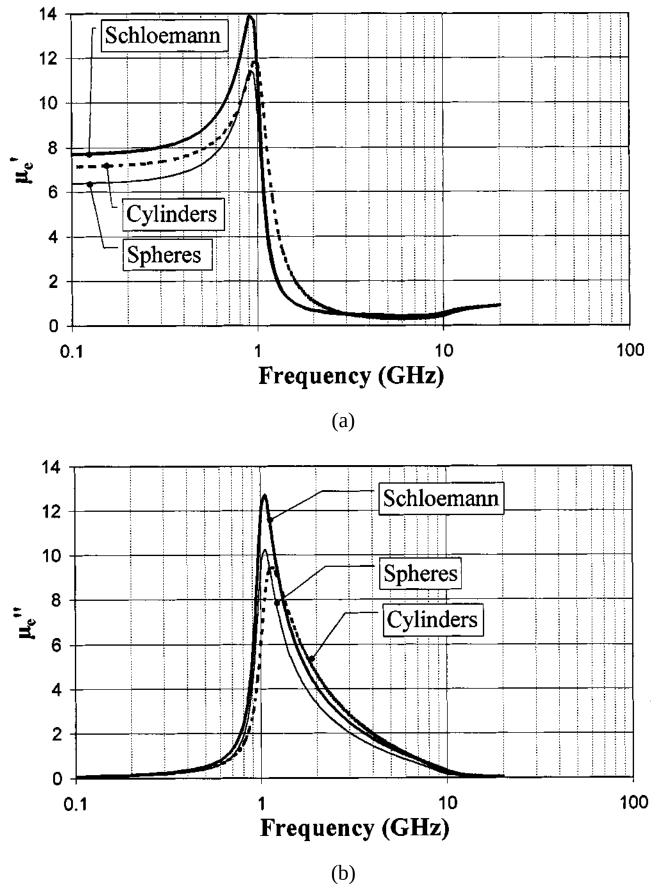

Fig. 3. Comparison between the effective permeability of unmagnetized material, given by Schloemann's (17) and EMA for spherical and cylindrical domains. (a) Real part μ_e' and (b) imaginary part μ_e''.

A. The Demagnetized State

In the demagnetized state, the domains are randomly oriented, the effective medium is isotropic, and the effective permeability tensor reduces to a scalar. In this case, (5) is used with $\theta_1 = 0$. Schloemann [3], [10] and Bouchaud and Zerah [4] have calculated the completely demagnetized state microwave permeability for an idealized domain model, made of "up" and "down" domains. The results obtained in [3] through exact calculations on a simple model and in [4] by using EMA are identical. The calculated permeability relates M_s, H_o, and ω through the μ and κ elements of the local permeability tensor. For the completely demagnetized state, the random orientation of the domains is taken into account in an approximate fashion by taking the average of the diagonal elements of their permeability tensor and gives

$$\mu_e^{\text{Sch}} = \frac{1 + 2\sqrt{\mu^2 - \kappa^2}}{3}. \qquad (17)$$

In comparison, using (5) and (13), we get the following equations to solve:

$$5\mu_e^3 + (2\mu - 1)\,\mu_e^2 - \left[3\left(\mu^2 - \kappa^2\right) + 2\mu\right]\mu_e - \left(\mu^2 - \kappa^2\right) = 0 \qquad (18)$$

$$4\mu_e^3 - \mu_e\left[\left(\mu^2 - \kappa^2\right) + 2\mu\right] - \left(\mu^2 - \kappa^2\right) = 0. \qquad (19)$$

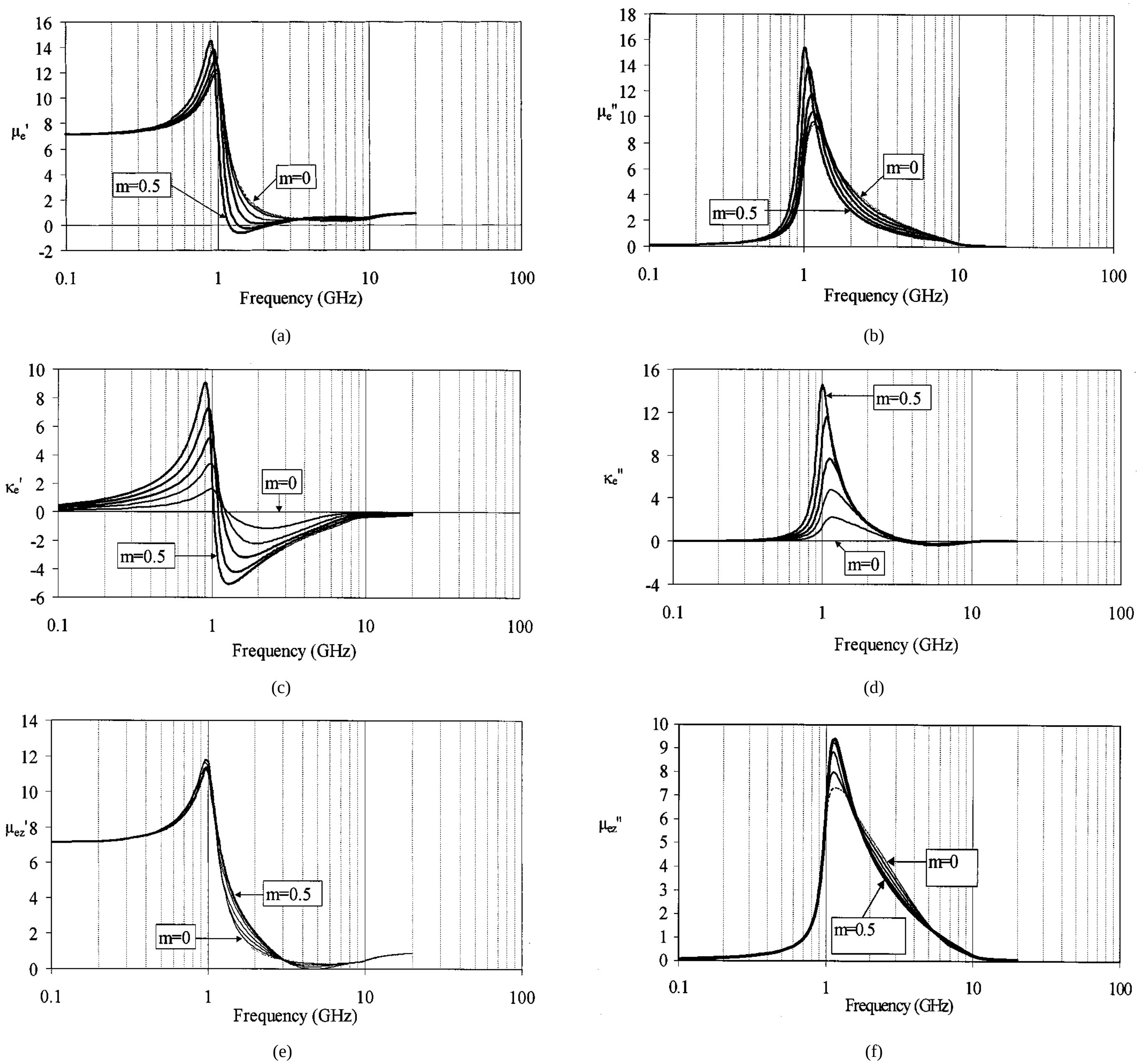

Fig. 4. Real and imaginary components of the effective permeability tensor versus frequency for m values from 0 to 0.5. (a) μ_e', (b) μ_e'', (c) κ_e', (d) κ_e'', (e) μ_{ez}', and (f) μ_{ez}''.

Equations (18) and (19) correspond to cylindrical and spherical domains, respectively. In Fig. 3(a) and (b), the real and imaginary parts of μ_e, as calculated from (18) and (19), are shown as functions of frequency, together with the results derived from Schloemann's theory [(17)]. We can notice that Polder–Smit-type domain resonance gives rise to a large loss (roughly in the band $[\omega_o, \omega_o + \omega_m]$, depending on α values) when ω_m/ω is near unity or greater. The main effect of changing the domain shape is to broaden the half-bandwidth spectrum when changing from cylinders to spheres. In spite of this, the permanence of losses up to $\omega_o + \omega_m$ still remains. In the static regime ($\kappa = 0$), (18) and (19) can be solved

$$\mu_e^{\text{cyl}} = \frac{3\mu + 1 + \sqrt{9\mu^2 + 26\mu + 1}}{10} \quad (20)$$

$$\mu_e^{\text{sph}} = \frac{\mu + \sqrt{\mu^2 + 8\mu}}{4} \quad (21)$$

while (17) reduces to

$$\mu_e^{\text{Sch}} = \frac{1}{3} + \frac{2}{3}\mu. \quad (22)$$

Equations (17) and (22) neglect some magnetostatic interactions in the averaging procedure. As a consequence, initial static permeabilities (linked to the gyromagnetic phenomenon only; dynamic domain walls displacements are not considered) obtained with (20) and (21) are always smaller than those obtained using (22), as can be seen in Fig. 3(a). It is interesting to note that (20)–(22) agree for $\mu = 1$. All of them predict that μ_e varies linearly with μ for $\mu \gg 1$, but the asymptotic slopes $d\mu_e/d\mu$ are different (0.6, 0.5, and 0.66, respectively).

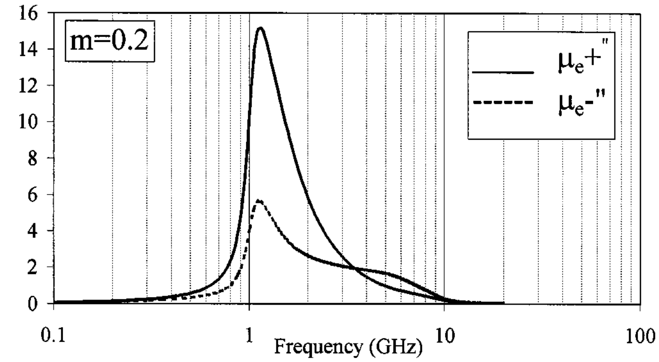

Fig. 5. Comparison in a partial magnetization state ($m = 0.2$) of the loss components μ_e+'' and μ_e-'' of the circularly polarized permeabilities.

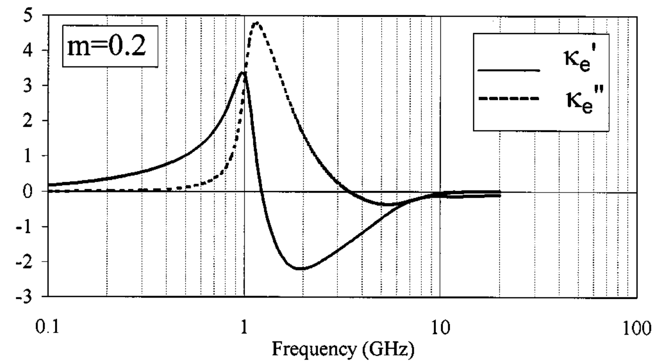

Fig. 6. Real and imaginary parts of the off-diagonal component κ_e in a partial magnetization state ($m = 0.2$).

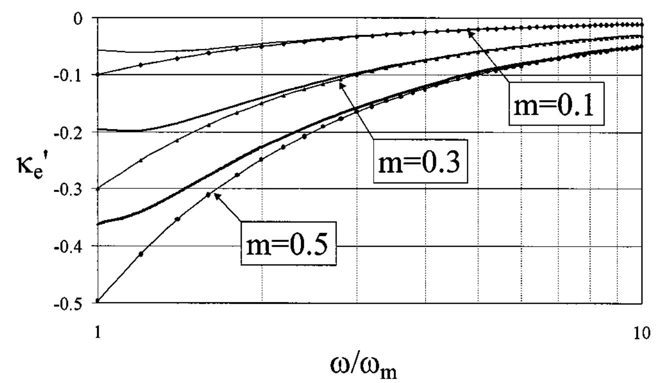

Fig. 7. Comparison between approximation for κ_e' given by (26) and EMA (cylindrical domains) for various magnetization states in the low-loss region.

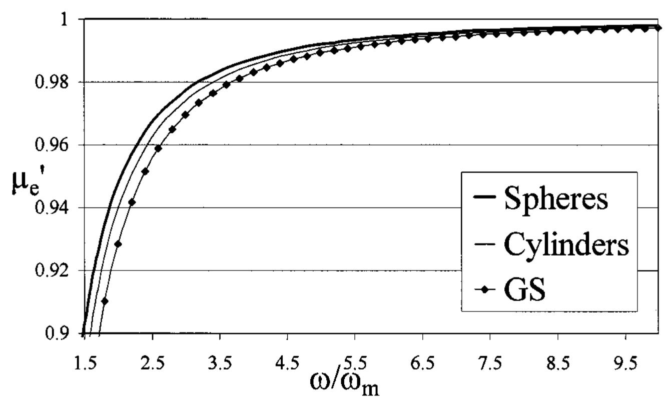

Fig. 8. Comparison between Green and Sandy's empirical formula [GS; (27)] and EMA for spherical and cylindrical domains for μ_e' component in partially magnetized state ($m = 0.5$).

B. The Partially Magnetized State

In Fig. 4(a)–(f), all the components of the permeability tensor have been obtained for different magnetization states (from $m = 0$ to 0.5 in steps of 0.1). These figures show how the magnetic spectrum changes when the relative magnetization is varied. For a fully magnetized material, the permeability is given by Polder's tensor. Fig. 4(a) shows that the μ_e' component has more resonant character for a magnetized material, as would be expected. As noted in [3] and [4], an antiresonance behavior is also found. It can be seen in Fig. 4(b), concerning the loss component μ_e'', that losses extend up to $\omega_o + \omega_m$ for all m values, but losses decrease in the high-frequency range of the loss region (and so half-bandwidth is sharper) when m is increasing. The most striking features comes from the off-diagonal component κ_e. Indeed, negative values for κ_e'' are obtained on a wide-frequency band (roughly 3–11 GHz). This can be made clearer using the circularly polarized unit vectors. In this case, the permeability tensor is diagonalized as follows:

$$\hat{\mu}_e = \begin{bmatrix} \mu_e+ & 0 & 0 \\ 0 & \mu_e- & 0 \\ 0 & 0 & \mu_{e_z} \end{bmatrix} \tag{23}$$

where $\mu_e+ = \mu_e + \kappa_e$ and $\mu_e- = \mu_e + \kappa_e$ refer to positive and negative circularly polarized fields, respectively. The physical requirement of a nonnegative energy loss in the ferrite leads to

$$\mu_e\pm'' \geq 0 \text{ and } \mu_{e_z}'' \geq 0. \tag{24}$$

From (24), it can be deduced that the only restriction imposed on κ_e'' is

$$|\kappa_e''| \leq \mu_e''. \tag{25}$$

A consequence of κ_e'''s being negative is that μ_e-'' is greater than μ_e+'' (see Figs. 5 and 6). This means that a magnetic field rotating oppositely to the precession is absorbed more than a field rotating with the precession (positive field). This is a different result from the completely magnetized state, where (as can be deduced from the Landau–Lifschitz equation of motion, which is usually assumed to be applied only to saturated media) μ_e+'' is always greater than μ_e-''. Those results have already been measured in [12] using a cavity resonator method, out of the loss region.

In the high-frequency range, relations have been previously proposed between the complex tensor components and some magnetic properties of the medium. Rado [6] proposed that the real part of the off-diagonal component κ_e' be given by

$$\kappa_e' = -m\frac{\omega_m}{\omega}. \tag{26}$$

The larger deviation from theory of κ_e' (Fig. 7) is in qualitative agreement with the greater departure from the approximation $\omega \gg \omega_m$. It has already been shown that this relationship is a reasonably good approximation for κ_e' as a function of m. Green and Sandy have shown experimentally the dependence of μ_e' and

μ'_{ez} for partially magnetized states fit to the following empirical equations:

$$\mu'_e = \mu'_o + (1-\mu'_o)\cdot m^{3/2} \tag{27}$$
$$\mu'_{e_z} = \mu_o'^{(1-m^{5/2})} \tag{28}$$

with

$$\mu'_o = \frac{1}{3} + \frac{2}{3}\sqrt{1-\left(\frac{\omega_m}{\omega}\right)^2}.$$

Good agreement can also be obtained with those empirical formulations. A comparison is given in Fig. 8 between (27) and the EMA with cylindrical and spherical domains for a magnetization state $m = 0.5$. We can see that μ'_e is weakly dependent on the magnetic domain shape and that comparison with empirical fit is very sharp.

IV. Conclusion

The model we propose allows the determination of all the components of the effective permeability tensor of polycrystalline ferrite in a single phase. The use of an optimization procedure is required anyway, due to the self-consistency character of the EMA. The model qualitatively accounts for many of the phenomena observed in partially magnetized ferrites at microwave frequencies. One can cite the onset of "low-field loss" near $\omega = \omega_m + \omega_o$ independently of the magnetic domain shape, as well as the variation of μ'_e, and μ''_e, in the demagnetized state, where the model agrees partly with Schloemann's formulation [10], the main difference coming from the static initial permeability. In the partially magnetized state, comparisons have been made with empirical formulas in the low-loss region. Good agreement between the calculated behavior and the empirical formulas was obtained. In the lossy region, some characteristics already measured on partially magnetized materials have been put forward—in particular, negative values of κ''_e in a wide frequency region. The domain shape and orientation have been tackled with more flexibility than in the previous models [3], [4], [10]. Results presented for uniaxial anisotropy can also be extended to cubic anisotropy, through suitable expression for the distribution function f [(5)]. The action of an external magnetizing static field has been studied. This paper does not take account of porosity or nonmagnetic volume fractions. We believe that these topics can be addressed with the techniques described in this paper, and we intend to make them the subject of a future publication.

Appendix

The purpose of this Appendix is to link tensor $\hat{\Gamma}$ to tensor $\hat{N}$ [13]. Inside the medium, the following equations have to be satisfied:

$$b = \mu_0\cdot[h+m] \tag{A-1}$$
$$\nabla\wedge h = 0 \tag{A-2}$$
$$h = -\nabla V \tag{A-3}$$
$$\nabla\cdot b = 0 \tag{A-4}$$

where μ_o is the permeability of vacuum. We have

$$\nabla^2 V(r) = \nabla\cdot m(r). \tag{A-5}$$

Equation (A-5) can be formally solved

$$\begin{aligned} V(r) &= \int_V -G_o(r,r')\nabla'\cdot m(r')d^3r' \\ &= -\int_V m(r')\cdot\nabla' G_o(r,r')d^3r \end{aligned} \tag{A-6}$$
$$\nabla^2 G_0 = -\delta(r-r'). \tag{A-7}$$

We get the field inside the medium

$$\begin{aligned} h(r) &= -\int_V \left[m(r')\cdot\nabla'\right]\nabla G_o(r,r')d^3r' \\ &= -\left[\int_V \hat{\Omega}_o\cdot d^3r'\right] m(r') = -\hat{N}m(r'). \end{aligned} \tag{A-8}$$

Equation (A-8) allows definition of the shape tensor, in terms of Green's function G_o, as

$$\hat{N} = \int_V \hat{\Omega}_o\cdot d^3r' \tag{A-9}$$
$$\Omega_o^{\beta\gamma} = -\frac{\partial}{\partial x_\beta}\frac{\partial}{\partial x'_\gamma}G_o(r-r'). \tag{A-10}$$

$\hat{\Gamma}$ is defined by

$$\Gamma^{\beta\gamma} = -\int_{V'} d^3x'\cdot\Omega_1^{\beta\gamma} \tag{A-11}$$
$$\Omega_1^{\beta\gamma} = -\frac{\partial}{\partial x_\beta}\frac{\partial}{\partial x'_\gamma}G_1(x-x') \tag{A-12}$$

with

$$\vec{\nabla}\cdot\left[\hat{\mu}_e\cdot\vec{\nabla}G_1(r-r')\right] = -\delta(r-r') \tag{A-13}$$
$$G_1(r-r') = 0 \text{ on } S. \tag{A-14 }$$

In anisotropic ferrimagnetic medium

$$\hat{\mu}_e = \begin{bmatrix} \mu_e & +j\kappa_e & 0 \\ -j\kappa_e & \mu_e & 0 \\ 0 & 0 & \mu_{e_z} \end{bmatrix}. \tag{A-15}$$

Equation (A-13) is written as

$$\left[\mu_e\frac{\partial^2}{\partial x^2} + \mu_e\frac{\partial^2}{\partial y^2} + \mu_{e_z}\frac{\partial^2}{\partial z^2}\right]\cdot G_1(r-r') = -\delta(r-r'). \tag{A-16}$$

In a rescaled coordinate system $R = (X, Y, Z)$ defined by

$$X = \frac{x}{\sqrt{\mu_e}},\ \ Y = \frac{y}{\sqrt{\mu_e}}\ \ Z = \frac{z}{\sqrt{\mu_{e_z}}} \tag{A-17}$$

we get

$$\left[\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2}\right]\cdot G_1(R-R') = -\frac{1}{\mu_e\cdot\sqrt{\mu_{e_z}}} \times\delta(R-R'). \tag{A-18}$$

A solution is given by

$$\mu_e\cdot\sqrt{\mu_{e_z}}\cdot G_1(R,R') = \frac{1}{4\pi}\frac{1}{||R-R'||} \tag{A-19}$$

i.e.,

$$G_1(r, r') = \frac{1}{4\pi\mu_e\sqrt{\mu_{e_z}}} \times \left(\frac{(x-x')^2}{\mu_e} + \frac{(y-y')^2}{\mu_e} + \frac{(z-z')^2}{\mu_{e_z}}\right)^{-1/2}. \tag{A-20}$$

Using (A-20), and comparing (A-9) and (A-11), we have

$$\Gamma^{\beta\gamma} = -\frac{1}{\sqrt{\mu_{e\beta}\cdot\mu_{e\gamma}}}\cdot N^{\beta\gamma} \tag{A-21}$$

where N is calculated in the rescaled coordinate system. In the isotropic case, (A-21) reduces to

$$\hat{\Gamma} = -\frac{\hat{N}}{\mu_e}. \tag{A-22}$$

REFERENCES

[1] D. Polder and J. Smit, "Resonance phenomena in ferrites," *Rev. Mod. Phys.*, vol. 25, pp. 89–90, Jan. 1953.

[2] J. J. Green and F. Sandy, "Microwave characterization of partially magnetized ferrites," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 641–645, 1974.

[3] E. Schloemann, "Microwave behavior of partially magnetized ferrites," *J. Appl. Phys.*, vol. 41, pp. 204–214, 1970.

[4] J. P. Bouchaud and P. G. Zérah, "Spontaneous resonances and universal behavior in ferrimagnets: Effective medium theory," *Phys. Rev. Lett.*, vol. 63, pp. 1000–1003, 1989.

[5] M. Igarashi and Y. Naito, "Tensor permeability of partially magnetized ferrites," *IEEE Trans. Magn.*, vol. 13, no. MAG-5, 1977.

[6] G. T. Rado, "Theory of the microwave permeability tensor and Faraday effect in nonsaturated ferromagnetic materials," *Phys. Rev.*, vol. 89, p. 529, 1953.

[7] P. Gelin and K. Berthou, "New consistent model for ferrite permeability tensor with arbitrary magnetization state," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 1185–1192, Aug. 1997.

[8] D. J. Bergman and D. Stroud, "Physical properties of macroscopically inhomogeneous media," *Solid State Phys.*, vol. 46, p. 147, 1992.

[9] D. Stroud, "Generalized effective-medium approach to the conductivity of an inhomogeneous material," *Phys. Rev. B*, vol. 12, no. 8, p. 3368, 1975.

[10] E. Schloemann, "Microwave ferrite materials," in *Wiley Encyclopedia of Electrical and Electronics Engineering*. New York: Wiley, 1999, vol. 13, pp. 103–107.

[11] S. Chikazumi, *Physics of Magnetism*. New York: Wiley, 1964.

[12] R. C. Le Craw and E. G. Spencer, "Tensor permeabilities of ferrites below magnetic saturation," in *IRE Conv. Rec.* New York, 1956, pt. 5, pp. 66–74.

[13] J. S. Ahn, K. H. Kim, T. W. Noh, D. H. Riu, K. H. Boo, and H. E. Kim, "Effective-medium theories for spheroidal particles randomly oriented on a plane: Application to the optical properties of a SiC whisker-Al_2O_3 composite," *Phys. Rev. B*, vol. 52, no. 31, 1995.

David Bariou was born in France in 1972. He received the M.S. and Ph.D. degrees in electronics from the University of Brest, France, in 1995 and 2000, respectively.

He is currently with DETI, Brest, France, working on microwave circulators. His research activities include the theoretical study and the characterization of heterogeneous magnetic materials and their application for microwave devices.

Patrick Quéffélec (M'99) was born in France in 1966. He received the Ph.D. degree in electronics from the University of Brest, France, in 1994.

Presently, he is Maître de Conférences at the University of Brest in the Laboratory for Electronics and Systems of Telecommunication. His research activities deal with the electromagnetic wave propagation in magnetic materials and the analysis of measurement methods for the microwave characterization of materials. Motivated by the applications of magnetic materials in nonreciprocal or tunable devices, he investigates the fundamental properties of ferromagnetic composites at microwave.

Philippe Gelin was born in France in 1948. He received the Ph.D. degree in physics from the Technical University of Lille, France, in 1981.

He is currently a Professor of electrical engineering at the Ecole Nationale Supérieure des Télécommunications de Bretagne, Brest, France. His research interests include wave–matter interactions and the modeling and characterization of materials. He is a member of the Laboratory for Electronics and Systems of Telecommunication.

Marcel Le Floc'h was born in France in 1945. He received the Docteur-és-Sciences degree from the Université de Bretagne Occidentale, France, in 1983.

He is presently a Professor in the Electronic Department of the Université de Bretagne Occidentale and a Member of the Laboratory for Electronics and Systems of Telecommunication.