ESTIMATION OF THE SPREADING WAVEFORM OF A SPREAD SPECTRUM TRANSMISSION

Gilles BUREL and Céline BOUDER

L.E.S.T. - UMR CNRS 6616
6 avenue Le Gorgeu, 29285 BREST cedex, FRANCE
e-mail: Gilles.Burel@univ-brest.fr
Celine.Bouder@univ-brest.fr

ABSTRACT

Spread spectrum signals have been used for secure communications for several decades [4]. Nowadays, they are also widely used outside the military domain, especially in Code Division Multiple Access (CDMA) systems [2]. Due to their low probability of interception, these signals increase the difficulty of spectrum surveillance.
Direct-Sequence Spread Spectrum transmitters (DS-SS) use a periodical pseudo-random sequence to modulate the baseband signal before transmission. In the context of spectrum surveillance, the pseudo-random sequence used by the transmitter is unknown (as well as other transmitter parameters such as duration of the sequence, symbol frequency and carrier frequency). Hence, in this context, a DS-SS transmission is very difficult to detect and demodulate, because it is often below the noise level.
In this paper, we propose a method for estimating the spreading waveform without prior knowledge on the transmitter. Only the period of the pseudo-random sequence is assumed to have been estimated (this can be done using cyclostationarity analysis). Our method is based on eigenanalysis techniques. We show that the spreading sequence can be recovered from the first and the second eigenvectors. This property provides a way to estimate the spreading sequence. Experimental results are given to illustrate the performances of the method and show that a good estimation can be obtained even when the signal is far below the noise level.

KEYWORDS

Digital Transmissions, Spread Spectrum, Estimation, Spreading Waveform, Spreading Sequence, Spectrum Surveillance

1. PRINCIPLE OF DIRECT-SEQUENCE SPREAD SPECTRUM TRANSMISSION

In direct-sequence spread spectrum (DS-SS), the information signal is modulated by a periodic pseudo-random sequence (PPRS) prior to transmission, resulting in a wideband signal with low probability of interception [2][4]. Indeed, the DS-SS signal can be transmitted below the noise level, because the receiver knows the pseudo-random sequence and therefore can use a correlator to increase the signal-to-noise ratio (SNR). For instance, with a pseudo-random sequence length equal to 31, the correlation gain is $15dB$. Hence, even if the DS-SS signal is received with SNR$=-5dB$, at the correlator output the SNR is +10 dB.
At the output of the receiver filter, the baseband signal is:

$$s(t) = \sum_{k=-\infty}^{+\infty} a_k h(t - kT_s) + b(t) \qquad (1)$$

where $h(t) = \sum_{k=0}^{P-1} c_k p(t - kT_c)$ is the spreading waveform..
$p(t)$ is the convolution of the transmission filter, the channel filter (which represents the channel echoes) and the receiver filter. $\{c_k , k=0..P-1 \}$ is the pseudo-random sequence, a_k is a symbol, T_s is the symbol period, T_c is the chip period ($T_s=P.T_c$), and $b(t)$ is the noise at the output of the receiver filter.

In the context of spectrum surveillance, the pseudo-random sequence used by the transmitter is unknown. In this paper, we propose a method to estimate the spreading waveform $h(t)$ without having to estimate the pseudo-random sequence $\{c_k , k=0..P-1 \}$. Furthermore, since the ultimate goal of spectrum surveillance is to estimate the symbols a_k , we can see from the equations above that when the received signal is correlated with $h(t)$ instead of with the pseudo-random sequence, a better estimation of the a_k is obtained.

In this paper, we assume that the symbol period T_s is known. It can be estimated by methods based on cyclostationarity analysis of the signal or, if the SNR is low, by methods based on fluctuations of correlation estimators [1]. All other parameters are unknown.

2. BLIND ESTIMATION OF THE SPREADING WAVEFORM

The received signal is sampled and divided into non-overlapping temporal windows, the duration of which is T_s. Let us note $\vec{s}$ the content of a window. The correlation matrix

$$R = E\left\{\vec{s}.\vec{s}^{\,H}\right\} \qquad (2)$$

can be estimated as follows: the N available vectors $\vec{s}$ can be placed in the columns of a matrix S (M rows, N columns), and the correlation matrix estimated by

$$\hat{R} = S.S^H / N \qquad (3)$$

The eigenanalysis of this matrix shows that there are two large eigenvalues. The reason is explained below.

Since the windows duration is equal to the symbol period, a window always contains the end of a symbol (for a duration $T_s - t_0$), followed by the beginning of the next symbol (for a duration t_0), where t_0 is unknown. Hence, we can write:

$$\vec{s} = a_m \vec{h}_0 + a_{m+1}\vec{h}_{-1} + \vec{b} \qquad (4)$$

$\vec{h}_0$ is a vector containing the end (duration $T_s - t_0$) of the spreading waveform $h(t)$, followed by zeros (duration t_0).

$\vec{h}_{-1}$ is a vector containing zeros (duration $T_s - t_0$) followed by the beginning (duration t_0) of the spreading waveform $h(t)$.

t_0 is the unknown desynchronization time between windows and symbols ($0 \le t_0 < T_s$).

The equations above show that:

$$R = E\left\{\left|a_m\right|^2\right\}\vec{h}_0.\vec{h}_0^H + E\left\{\left|a_{m+1}\right|^2\right\}\vec{h}_{-1}.\vec{h}_{-1}^H + \sigma^2 I \qquad (5)$$

where σ^2 is the noise variance. From this equation, we see that there are two large eigenvalues, $\sigma_a^2\left\|\vec{h}_0\right\|^2 + \sigma^2$ and $\sigma_a^2\left\|\vec{h}_{-1}\right\|^2 + \sigma^2$ (the order is unknown), where $\sigma_a^2 = E\left\{\left|a_k\right|^2\right\}$. These eigenvalues are associated to eigenvectors whose directions are given by $\vec{h}_0$ and $\vec{h}_{-1}$. All the other eigenvalues are σ^2.

The major remaining problem is to know how to extract information from the two first eigenvectors $\vec{u}_0$ and $\vec{u}_1$ in order to obtain the spreading waveform. Indeed, since matrix R can only be estimated from a limited number of signal samples, these vectors are noisy, and it is not trivial to recognize which one corresponds to $\vec{h}_{-1}$ and which one corresponds to $\vec{h}_0$. Furthermore, the desynchronization t_0 is unknown and there are unknown multiplicative factors between the eigenvectors $\vec{u}_i$ and the partial spreading vectors $\vec{h}_j$. The method we propose to solve this problem is as follows.

First of all, let us compute:

$$\vec{v}_i = S^T.\vec{u}_i^{\,*} \qquad (6)$$

It is obvious (see equation *(4)*) that the elements of $\vec{v}_i$ are estimates of the symbols (up to an unknown multiplicative factor). For clarity, let us assume that $\vec{u}_0$ corresponds to $\vec{h}_0$, and $\vec{u}_1$ to $\vec{h}_{-1}$ (generalization to the other case is trivial). Let us note $\vec{u}_i = z_i.\vec{h}_{-i}$, where z_i are unknown complex numbers. In this case, equations *(4)* and *(6)* show that the elements of $\vec{v}_i$ are estimates of a_i / z_i , a_{1+i} / z_i , ..., a_{m+i} / z_i , ..., a_{N-1+i} / z_i. Let us compute:

$$\omega_{ij} = \sum_{m=1}^{N-1} v_0(m-i)^*.v_1(m-j) \qquad (7)$$

It is clear that, here, we have $\omega_{01} > \omega_{10}$, because it is only in ω_{01} that the estimated symbols are shifted in a coherent way. For the other possible correspondence, we would obtain $\omega_{10} > \omega_{01}$. Hence, the relative values of the ω_{ij} can be used to determine which correspondence is the right one.

Now, let us assume that the correspondence mentioned above has been determined as being the right one (once again, generalization to the other case is trivial). We build the 2M-dimensional vector below:

$$\vec{u} = \begin{bmatrix} \vec{u}_1 \\ \frac{z_1}{z_0}\vec{u}_0 \end{bmatrix} \qquad (8)$$

Obviously, we have:

$$\vec{u} = z_1 \begin{bmatrix} \vec{h}_{-1} \\ \vec{h}_0 \end{bmatrix} \qquad (9)$$

Finally, the estimated spreading waveform is obtained by extracting from $\vec{u}$ the M-dimensional subvector with the highest norm.
The spreading waveform is estimated up to an unknown multiplicative factor. This is not a problem because in any transmission system, the channel is modeled by at least an unknown multiplicative factor. Hence, this uncertainty is also included in the channel model. This is why transmission systems use either differential coding, or periodically occurring known symbol sequences, in order to remove this uncertainty.

Let us come back to equation *(8)*. To apply this equation, we need an estimate of z_1 / z_0. It can be obtained as explained below.
Considering that the elements of $\vec{v}_i$ are estimates of a_i / z_i, a_{1+i} / z_i, ..., a_{m+i} / z_i, ..., a_{N-1+i} / z_i, we can write:

$$\|\vec{v}_i\|^2 \approx |z_i|^{-2}.\|\vec{a}\|^2 + N.\sigma^2 \qquad (10)$$

where $\vec{a}$ is a vector containing the symbols. Hence:

$$\left|\frac{z_1}{z_0}\right|^2 \approx \frac{\|\vec{v}_0\|^2 - N\sigma^2}{\|\vec{v}_1\|^2 - N\sigma^2} \qquad (11)$$

and σ^2 can be estimated by averaging the eigenvalues numbered 3 to *M*.
Having obtained the modulus of z_1 / z_0, we can now estimate its phase. From equation *(7)* we see that:

$$\omega_{01} \approx \frac{1}{z_0^* . z_1} \sum_{m=1}^{N-1} |a_m|^2 \qquad (12)$$

Hence we can write:

$$Arg(z_1 / z_0) \approx -Arg(\omega_{01}) \qquad (13)$$

3. EXPERIMENTAL RESULTS

To illustrate the approach, a DS-SS signal is generated using a random-sequence of length 31 (it is one of the Gold Sequences [3] which are traditionally used in CDMA systems). The symbols belong to a QPSK constellation (Quadrature Phase Shift Keying). The SNR is $-8dB$ (hence, the noise power in the signal passband is more than 6 times the signal power). 211 windows were used for estimating the correlation matrix. To simplify the interpretation of illustrations, the sampling period was chosen equal to $T_s /31$ (of course, in practice, this is not a requirement. The only requirement is to have a sampling frequency large enough with respect to receiver filter bandwidth).

Figure 1 shows the eigenvalues: we can clearly distinguish the first and second eigenvalues. The estimated spreading waveform obtained from the corresponding eigenvectors is shown. Basically, the estimation is composed of complex numbers. Here, the estimation has been projected on its principal axis in the complex plane, for easier comparison with the true sequence. The comparison with the true spreading waveform shows that a good estimation is obtained, even with negative SNR.

Here, the spreading sequence was real, but the method can also deal with complex spreading sequences, because nowhere in the method development we assumed that the sequence was real. While complex spreading sequences are not yet widely used in current spread spectrum systems, the situation may change in a close future.

A classical spread spectrum receiver correlates the spreading sequence with the received signal in order to estimate the symbols. If we feed a classical receiver with the waveform estimated by our approach instead of the true sequence, the difference is extremely low: the cosinus of the angle between the vectors (true and estimated waveforms) is 0.983, which means that they are almost aligned, hence they provide almost the same correlation results.

4. CONCLUSION

A method for blind estimation of the spreading waveform of a direct-sequence spread spectrum transmission has been proposed. The method is based on eigenanalysis techniques. After showing that the two first eigenvectors of the received signal correlation matrix contain information about the spreading waveform, we detailed the proposed approach to actually extract this information, and build an estimate of the spreading waveform. Experimental results show that a good estimate can be obtained even for very low SNR.

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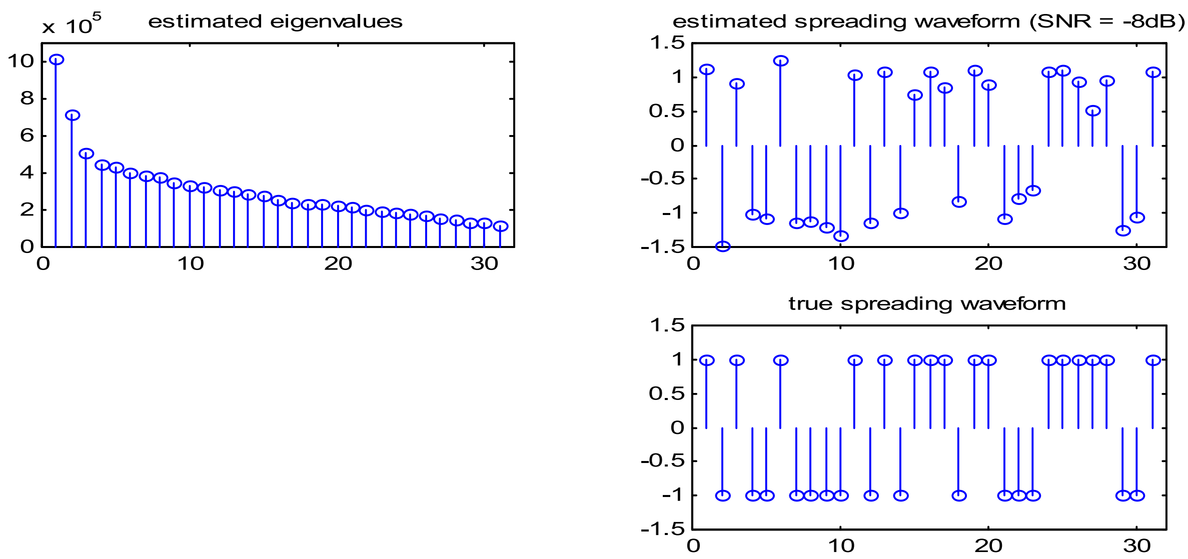

Figure 1: Experimental Results