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Minimum BER Diagonal Precoder for MIMO Digital Transmissions

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We propose a Minimum Bit Error Rate (MBER) diagonal Precoder for Multi-Input Multi-Output (MIMO) transmission systems. This work is based on previous results obtained by Sampath *et al.* [1] in which the global transmission system (precoder and equalizer) is optimized with the Minimum Mean Square Error (MMSE) criterion. This process leads to an interesting diagonality property which decouples the MIMO channel into parallel and independent data streams and allows to perform an easy ML detection. This system is then optimized using a new diagonal precoder that minimizes the BER. Our work is motivated by the fact that, from a practical point of view, people are likely to prefer a system that minimizes the BER rather than the Mean Square Error. The performance improvement is illustrated via Monte Carlo simulations using a Quadratic Amplitude Modulation (QAM).

1. Introduction

Multi-Input Multi-Output (MIMO) digital transmission systems currently retain more and more attention due to the very high spectral efficiencies they can achieve. Most existing systems such as spatial multiplexing [6] or space-time coding [5] assume no channel knowledge at the transmitter. However, in many wireless applications, feedback does exist (e.g., symmetric or asymmetric duplex transmissions), and channel information can be made available at the transmitter. Indeed, only a very small data rate is used to provide channel information to the transmitter.

The question, then, is how to take profit of this information to globally optimize the transmission system. In [1,2] Sampath *et al.* designed a jointly optimum linear precoder and equalizer for MIMO systems according to the Minimum Mean Square Error (MMSE) criterion.

In this article, using the Maximum Likelihood detection, we propose a new precoder which minimizes the Bit Error Rate (MBER) instead of the Mean Square Error. In a first step, our approach takes profit of an interesting property of the system proposed in [1]: the precoder and equalizer which minimize the MMSE criterion appear to diagonalize the global transmission system. Hence, we can express the BER in a quite simple way. Then, using Lagrange multipliers, we derive the coefficients of the precoder which provides the minimum BER, under the constraint of a given transmission power. This second step guarantees minimization of the overall BER of the diagonalized system, independently on the way the channel was diagonalized.

The paper is organized as follows. In Section 2, we introduce the system model and we show that the system proposed in [1] is able to transform a MIMO system into independent virtual data streams. Then, in Section 3, we explain how to derive the precoder which minimizes the BER. Simulation results are provided in Section 4. Finally, a conclusion is drawn in Section 5.

2. An interesting property of MMSE-optimized MIMO system

Consider a MIMO system with n_r receive and n_t transmit antennas, over which we want to send b independent data streams (see Fig.1). The system equation is:

$$\boldsymbol{y} = GHF\boldsymbol{s} + G\boldsymbol{n} \quad (1)$$

where H is an $(n_r \times n_t)$ channel matrix, F an $(n_t \times b)$ precoder matrix and G a $(b \times n_r)$ equalizer matrix. $\boldsymbol{s}$ is the $(b \times 1)$ transmitted vector of symbols and $\boldsymbol{n}$ is the $(n_r \times 1)$ noise vector. We assume

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that channel information can be made available at the transmitter side.

We assume[1]: $E\{\boldsymbol{ss}^*\} = I$, $E\{\boldsymbol{nn}^*\} = R_n$ and $E\{\boldsymbol{sn}^*\} = 0$.

Let us note k the rank of H. Obviously, we must have $1 \leq b \leq k \leq \min(n_t, n_r)$.

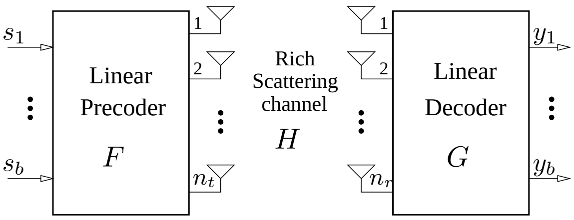

Figure 1. MIMO system

In [1], Sampath *et al.* designed F and G matrices to minimize any weighted sum of symbol estimation errors, subject to the constraint $trace\{FF^*\} = p_0$, where p_0 is the available transmission power.

We will take profit of one property of their approach, which is the fact that the optimum matrices decouple the MIMO channel into b parallel and independent data streams[2]. Indeed, if we note D the matrix which links the global system output to the system input, and R_{n_v} the covariance matrix of the noise affecting the global output, we have:

$$D = GHF = \Phi_g \Lambda \Phi_f \tag{2}$$

$$R_{n_v} = GR_nG^* = \Phi_g \Lambda \Phi_g^* \tag{3}$$

where Φ_f, Φ_g and Λ are $b \times b$ diagonal matrices with real non-negative elements. These equations clearly show that D and R_{n_v} are diagonal matrices, hence the MIMO system can be considered as a set of independent virtual data streams.

The procedure to compute Φ_f, Φ_g and Λ is described in [1]. Eq. (2) directly derives from Lemma 2 in [1] and Eq. (3) is straightforward by using (14) and (12) in [1].

Using the optimum MMSE precoder and equalizer, the transmission model (corresponding to right dashed box in Fig. 2) for sub-channel number i is then:

$$y_i = d_i s_i + n_{vi} \tag{4}$$

where d_i is the $(i,i)^{th}$ entry of diagonal matrix D given by Eq. (2), and n_{vi} is the $(i)^{th}$ component of noise vector $\boldsymbol{n}_v$. The covariance matrix of this noise vector is $R_{n_v} = E[\boldsymbol{n}_v \boldsymbol{n}_v^*] = diag\{\sigma_i^2\}_{i=1}^b$ given by Eq. (3).

3. Precoder optimization for minimum BER

In this section, we assume that the symbol decision is performed with the ML criterion. The ML implementation is trivial, because the transmission system can be seen as b parallel and independent data streams, as explained in the previous section (hence the decision reduces to taking the nearest-neighbor in the constellation). For clarity of presentation, we assume that the symbols belong to an M-QAM constellation. The approach can be adapted to any other constellation for which an expression of the BER is available.

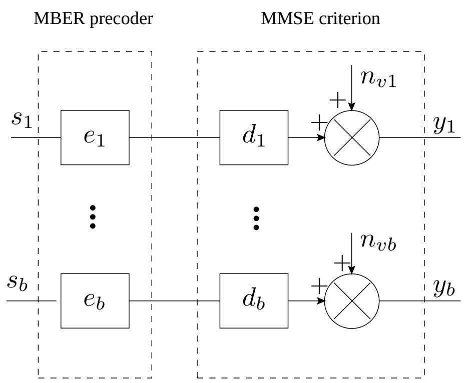

Figure 2. Equivalent MIMO system with an additive diagonal precoder E

We propose to add a diagonal precoder E (the resultant precoder F_r is then $F_r = FE$) in order

[1] The symbol $*$ denotes transpose conjugate.
[2] This property was not searched for by the authors of [1]: it just appears to be an interesting consequence of the MMSE criterion optimization.

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to minimize the BER (see Fig. 2). Please note that the extra cost of this precoder is negligible because *i)* it is diagonal and *ii)* it can be included in the global precoder matrix.

We will now derive a closed form solution for the optimum precoder E that minimizes the BER of the transmission. Since virtual channel matrix D and output noise covariance matrix R_{n_v} are diagonal, we restrict our search to a diagonal matrix $E = diag\{e_i\}_{i=1}^{b}$. The transmission model (see Fig. 2) for sub-channel number i is then:

$$y_i = d_i e_i s_i + n_{vi} \tag{5}$$

Note that the elements d_i are real and positive, hence we will choose real and positive values for the e_i (it is easy to show that considering negative or complex values would not modify the signal to noise ratio (SNR), hence would not improve the BER). The SNR for each sub-channel is:

$$\rho_i = \frac{d_i^2 e_i^2}{\sigma_i^2} \tag{6}$$

For a square M-QAM constellation, the BER is $P_e = \frac{1}{b}\sum_{i=1}^{b} P_{e,i}$ where the BER in sub-channel i is [4]:

$$P_{e,i} = \alpha_M \times \operatorname{erfc}\sqrt{\beta_M \times \rho_i} \tag{7}$$

with $\alpha_M = \frac{2}{\log_2 M}(1 - \frac{1}{\sqrt{M}})$ and $\beta_M = \frac{3}{2(M-1)}$.

In order to keep the same total transmission power p_0 as previously, we determine the e_i which minimize P_e, under the constraint $trace(FEE^*F^*) = p_0$. This constraint is equivalent to:

$$\sum_{i=1}^{b} c_i\, e_i^2 = p_0 \tag{8}$$

with $c_i = \sum_{j=1}^{n_t} |F_{ji}|^2$ (F_{ji} is the $(j,i)^{th}$ entry of F). Using Lagrange multiplier μ, the criterion to optimize is:

$$C = \frac{\alpha_M}{b}\sum_{i=1}^{b} \operatorname{erfc}\left(\frac{\sqrt{\beta_M} d_i e_i}{\sigma_i}\right) + \mu\left(\left(\sum_{i=1}^{b} c_i\, e_i^2\right) - p_0\right) \tag{9}$$

Cancellation of $\frac{\partial C}{\partial e_i}$ provides:

$$e_i^2 = \frac{\sigma_i^2}{2\beta_M d_i^2} W_0\left(\frac{2 d_i^4 \alpha_M^2 \beta_M^2}{\mu^2 \pi \sigma_i^4 b^2 c_i^2}\right) \tag{10}$$

where W_0 stands for Lambert's W function of index 0 [3]. This function $W_0(x)$ is an increasing function. It is positive for $x > 0$, and $W_0(0) = 0$. Hence, when μ^2 increases, the e_i^2 decrease. Therefore, μ^2 can be easily determined, using the constraint (8).

Let us summarize the proposed method. First, given channel matrix H and noise covariance matrix R_n, we compute the precoder F and the equalizer G using the MMSE criterion under the constraint $trace(FF^*) = p_0$, as described in [1]. Second, a new diagonal precoder E is added. The diagonal elements of this precoder are given by Eq. (10) where μ is determined using constraint (8).

4. Simulation results

For our Monte-Carlo simulations, we use the same configuration as in [1], that is $n_t = 5$ transmitters, $n_r = 5$ receivers, and we transmit 4 independent data streams over the system. A 4-QAM constellation is used. For each SNR, $10,000,000$ vectors of 4 symbols are transmitted during the Monte-Carlo simulations. Every $1,000$ transmitted vectors, a new H and a new R_n are randomly chosen, in order to obtain results that do not depend on a particular channel, nor on particular noise statistics. Entries of H are i.i.d. zero-mean unit-variance complex Gaussian random variables. Matrices R_n are obtained by $R_n = T_n T_n^*$ (where entries of T_n are i.i.d. zero-mean unit-variance complex Gaussian random variables) and then scaled according to the desired SNR. The SNR is defined as the ratio of the total transmitted power p_0 to the total received noise power. The ML detection is performed on the received symbols.

Fig. 3 represents the BER with respect to the SNR, for both systems: MBER and MMSE. The "theoretical" BER is numerically evaluated by averaging Eq. (7) over ρ_i for each realization of H and R_n. The additional precoder E obviously reduces the BER. We can also note that

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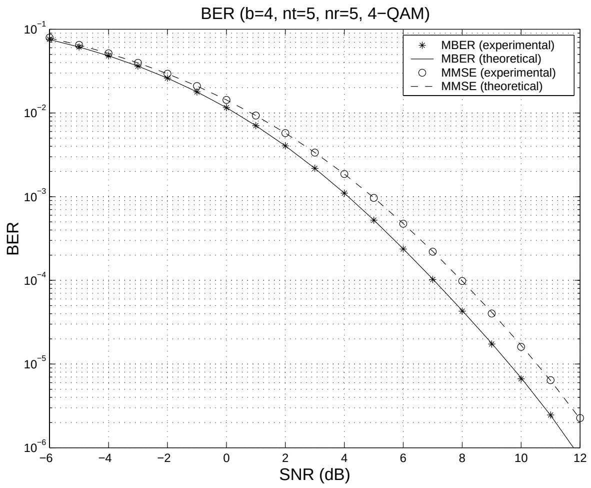

Figure 3. BER with respect to SNR

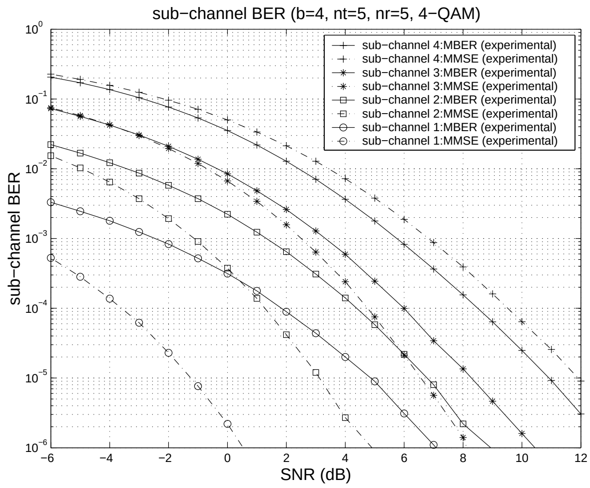

Figure 4. Sub-channel BER with respect to SNR

the improvement increases with the SNR. For large SNR, the MBER precoder provides an improvement about a factor of 2.5 over the MMSE precoder, for a negligible additional computational cost. In order to illustrate the impact of the MBER precoder on each sub-channel, Fig. 4 shows the sub-channels BERs for both systems. We observe that the BER improvement is focused only on the less favored sub-channel (symbol '+' on Fig. 4).

5. Conclusion

In this paper, we derived the optimum precoder for MIMO channels under the Minimum Bit Error Rate criterion. Sampath *et al.* proposed in [1] a joint transmit and receive optimization scheme for a MMSE criterion. Thanks to the diagonality of the obtained global system a new diagonal precoder can be designed which guarantees better results in term of BER. In order to show the efficiency of our method, we have made comparisons between MIMO systems with the MMSE precoder and the proposed MBER precoder through Monte-Carlo simulations. This kind of transmission scheme is directly applicable to narrowband wireless channels, in which channel information can be made available at the transmitter.

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