COMPARISON OF d_{min}-BASED PRECODER WITH OSTBC AND MAXIMUM SNR-BASED PRECODER FOR MIMO SYSTEMS

Baptiste Vrigneau, Philippe Rostaing and Gilles Burel
LEST-UMR CNRS 6165, 6 Av. Le Gorgeu, CS 93837
29238 Brest Cedex 3, France
Firstname.Lastname@univ-brest.fr (fax : 33298016395)

Abstract: In order to compare the performances of three transmission schemes of key-interest able to reach the maximum diversity order, guarantee of robust transmission, we used a dual-polarization channel model to take into account polarization diversity and transmit/receive correlation. These schemes under study were *i*)two systems using full channel state information (CSI) at the transmitter side (Tx-CSI), *i.e.* the $d_{\min}$-based precoder (maximization of minimum Euclidean distance ($d_{\min}$)) and the beamforming or max-SNR precoder (maximization of the output signal-to-noise ratio) and *ii*) the Alamouti scheme belonging to the orthogonal space-time block code (OSTBC) family which needs no Tx-CSI. Their performances were also compared with imperfect CSI to determine the gain in dB due to the use of Tx-CSI . This study was extended to several dual-polarized antennas.

Keyword: Alamouti scheme, $d_{\min}$ based precoder, beamforming, correlated Rayleigh and Rice fading channel, dual-polarization channel.

1 INTRODUCTION

Further to the constant evolution of wireless communications, the use of Multiple-Input Multiple-Output (MIMO) systems may enable one to improve a communication link within a scattering environment like an indoor WAN [4] by increasing the spectral efficiency or the transmission robustness. For example, spatial multiplexing (SM) is a simple solution to significantly improve the spectral efficiency by demultiplexing different symbols over the transmit antennas [3] [8]. Otherwise, the transmission reliability can be improved by achieving the maximum spatial diversity order with the orthogonal space-time block code (OSTBC) and max-SNR precoder. These schemes rely on two different strategies: the first one introduces redundancy by transmitting several symbol combinations through the channel over several symbol periods [1] [7]; this solution needs no CSI at the transmitter side (Tx-CSI), but decreases the spectral efficiency in comparison to SM. In the second solution, where the symbols are transmitted along the most favorable channel direction, the received SNR is maximized [6]; it thus implies a precoder with Tx-CSI; moreover, it uses only one symbol during one symbol period over the transmit antennas, and then decreases the spectral efficiency in comparison to SM. A recent paper [2] about a new precoder based on the maximizing of the minimum Euclidean distance between two received symbols, $d_{\min}$, demonstrated its ability to improve both spectral efficiency (same bit-rate as the SM) and transmission robustness; but, alike the max-SNR precoder, it requires the knowledge of Tx-CSI.

Another report [5] described the channel model of a system with a single antenna structure to use two orthogonal polarizations. This model takes into account the transmit and receive correlation as well as the antenna ability to discriminate among polarizations. These parameters, which depend on antenna characteristics and scattering environment, strongly affect the system performance. As the above solutions reach the maximum diversity gain, we wondered about the benefits of Tx-CSI upon the system performances in term of bit-error probability. Furthermore, the knowledge of CSI being paramount, it drove us to study robustness with respect to channel estimation errors. We also studied a straightforward extension for multiple dual-polarized antennas.

So, in this report, section 2 will introduce the dual-polarization channel model. Section 3 will briefly describe the three schemes (OSTBC, $d_{\min}$-based and max-SNR precoders), and section 4 will compare simulations through the dual-polarization channel. Finally, our conclusion will be drawn in Section 5.

2 CHANNEL MODEL

In this study, the channel model for wireless links employing dual-polarized antennas takes into account cross-polarization discrimination (XPD), Ricean K-factor and fading signal correlation [5]. The system employs one dual-polarized transmit antenna and one dual-polarized receive antenna with slanted polarization ($+45°/-45°$). Since each polarization mode can be treated as a separate channel, this scheme equivalent to a MIMO system with two transmit antennas and two receive antennas requires only one physical transmit and one physical receive antenna. This constitutes a beneficial and promising cost-and-space-effective solution.

The elements of $\mathbf{H} = \begin{pmatrix} h_{0,0} & h_{0,1} \\ h_{1,0} & h_{1,1} \end{pmatrix}$ are complex and usually correlated Gaussian random variables; the channel is decomposed into a fixed component and a variable one as follows:

$$\mathbf{H} = \sqrt{\frac{K}{1+K}}\overline{\mathbf{H}} + \sqrt{\frac{1}{1+K}}\widetilde{\mathbf{H}} \tag{1}$$

where $E\{\mathbf{H}\} = \sqrt{\frac{K}{1+K}}\overline{\mathbf{H}}$ and $\sqrt{\frac{1}{1+K}}\widetilde{\mathbf{H}}$ are respectively the average and variable components of the channel matrix, and K is the Ricean factor defined as the ratio of the power in the fixed component to the power in the variable component. The elements of the matrix $\overline{\mathbf{H}} = [\overline{h}_{i,j}]$ are fixed and satisfy $|\overline{h}_{0,0}|^2 = |\overline{h}_{1,1}|^2 = 1$ and $|\overline{h}_{0,1}|^2 = |\overline{h}_{1,0}|^2 = \alpha_f$ where $0 \leq \alpha_f \leq 1$ depends on the XPD of the fixed component. In addition, the elements of $\widetilde{\mathbf{H}} = [\widetilde{h}_{i,j}]$ are centered random variables with the following properties:

$$E\{|\widetilde{h}_{0,0}|^2\} \;=\; E\{|\widetilde{h}_{1,1}|^2\} \;=\; 1, \qquad E\{|\widetilde{h}_{0,1}|^2\} \;=\; E\{|\widetilde{h}_{1,0}|^2\} \;=\; \alpha \tag{2}$$

where $0 \leq \alpha \leq 1$ depends on the XPD for the variable component of the channel and scattering-induced coupling between orthogonal polarizations. It is worth recalling that, when α is small, discrimination of orthogonal polarizations is good. The correlation of $\mathbf{H}$ elements is defined as follows:

$$t = \frac{E\{\widetilde{h}_{0,0}\widetilde{h}_{0,1}^*\}}{\sqrt{\alpha}} = \frac{E\{\widetilde{h}_{1,0}\widetilde{h}_{1,1}^*\}}{\sqrt{\alpha}}, \qquad r = \frac{E\{\widetilde{h}_{0,0}\widetilde{h}_{1,0}^*\}}{\sqrt{\alpha}} = \frac{E\{\widetilde{h}_{0,1}\widetilde{h}_{1,1}^*\}}{\sqrt{\alpha}} \tag{3}$$

where t denotes the transmit correlation coefficient, and r is the receive correlation coefficient. Let us set that $E\{\widetilde{h}_{0,0}\widetilde{h}_{1,1}^*\} = E\{\widetilde{h}_{1,0}\widetilde{h}_{0,1}^*\} = 0$. Under these assumptions, the correlation matrix can be expressed as:

$$\mathbf{R_H} = E\{\mathrm{vect}\{\mathbf{H}\}\,\mathrm{vect}\{\mathbf{H}\}^*\} = \begin{pmatrix} 1 & r\sqrt{\alpha} & t\sqrt{\alpha} & 0 \\ r\sqrt{\alpha} & \alpha & 0 & t\sqrt{\alpha} \\ t\sqrt{\alpha} & 0 & \alpha & r\sqrt{\alpha} \\ 0 & t\sqrt{\alpha} & r\sqrt{\alpha} & 1 \end{pmatrix}$$

where vect($\mathbf{H}$) is a vector of length 4 obtained by stacking each column of the matrix $\mathbf{H}$.

3 DESCRIPTION OF THE DIFFERENT SCHEMES

An overview of the different schemes, OSTBC, $d_{\min}$ and max-SNR precoders, is presented in this section for an arbitrary number of n_T transmit and n_R receive antennas.

3.1 OSTBC

Figure 1 illustrates an OSTBC design transmission. On assuming a stationary channel during N_p symbol periods, an input stream of N_s symbols is transmitted to the n_T transmit antennas through the orthogonal code matrix $\mathbf{C}$ $[n_T \times N_p]$. One should note that the OSTBC uses N_p symbol periods and that the matrix $\mathbf{C}$ is a concatenation of N_p vectors. It, thus, affects the transmission rate by $R = N_s/N_p$ and delays the estimation by N_p symbol periods. The best transmission rate, $R = 1$, is got by using the Alamouti code. The other available codes imply $R = 3/4$ for $n_T = 3$ or $n_T = 4$ and $R = 1/2$ for any n_T for complex symbols. The material configuration imposes the code to be used; here, in the case of one transmit and one receive dual-polarized antennas, it is the Alamouti code defined by:

$$C = \begin{pmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{pmatrix} \quad (4)$$

By assuming the symbols s_i normalized so that $E[|s_i|^2] = 1$, the input-output relation is:

$$\mathbf{Y} = \sqrt{E_T/n_T}\,\mathbf{H}\mathbf{C} + \mathbf{N} \quad (5)$$

where E_T is the total transmit average energy over a symbol period, $\mathbf{H} = [h_{i,j}]_{i,j=1}^{n_R,n_T}$ $[n_R \times n_T]$ is the channel matrix where $h_{i,j}$ is a complex gain from the j^{th} transmit antenna to the i^{th} receive one, $\mathbf{Y}$ $[n_R \times N_p]$ is the received samples matrix and $\mathbf{N}$ $[n_R \times N_p]$ is the matrix of additive and temporally i.i.d. Gaussian zero-mean σ_n^2-variance random noise samples. The channel matrix is estimated at the receiver side. The orthogonality of $\mathbf{C}$ allows one to simplify the receiver structure by recombining the received samples [1] with the channel complex gains to get the samples, $\tilde{s}_i$, which directly depend on the symbols s_i for $i = 1, \ldots, N_s$. A maximum likelihood (ML) detector enables one to estimate s_i:

$$\widehat{s}_i = \min_{s_l \in \mathcal{C}} \left\{ \|\widetilde{s}_i - \sqrt{E_T/n_T}\|\mathbf{H}\|_F^2 s_l\|^2 \right\} \quad (6)$$

for $i = 1, \ldots, N_s$, $\|.\|_F^2 = trace(\mathbf{H}\mathbf{H}^H) = trace(\mathbf{H}^H\mathbf{H})$ is the square Frobenius norm.

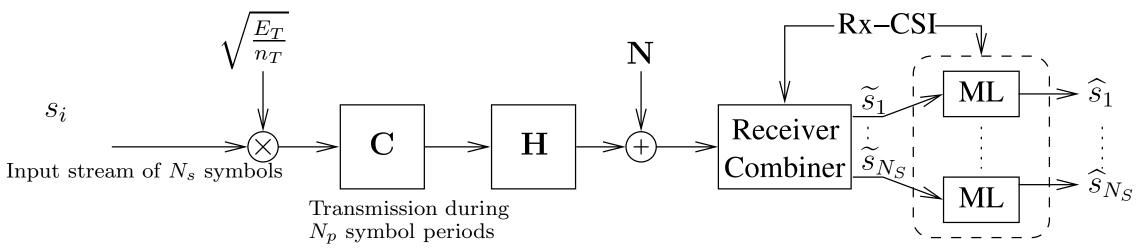


Figure 1: OSTBC transmission block diagram

The transmit scheme can be decoupled into N_s equivalent single-input single-output (SISO) systems as shown in Figure 2 with the gain $\sqrt{E_T/n_T\|H\|_F^2}$ and an additive complex Gaussian zero-mean and σ_n^2-variance noise sample n.

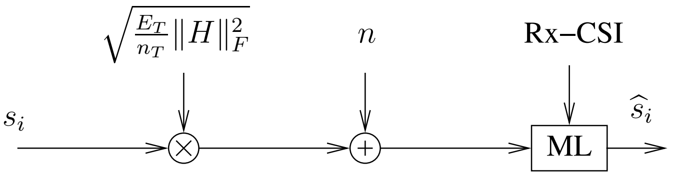


Figure 2: OSTBC equivalent SISO transmission block diagram

The next two subsections will deal with two transmissions schemes requiring CSI at the transmitter side.

3.2 $d_{\min}$-based precoder

The minimum Euclidean distance d_{min} between signal points at the receiver side affects the system performances, especially when an ML detector is used. The $d_{\min}$-based precoder, $\mathbf{F}$, is associated to a decoder, $\mathbf{G}$, at the receiver side and maximizes this criterion under the CSI condition at the transmitter side (Figure 3).

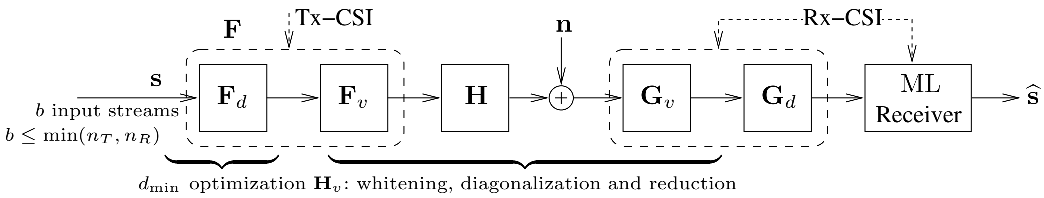


Figure 3: $d_{\min}$-based precoder transmission block diagram

In a first step, the transmit precoder, $\mathbf{F}$, is decomposed as $\mathbf{F} = \mathbf{F}_v\mathbf{F}_d$ and the receive decoder as $\mathbf{G} = \mathbf{G}_d\mathbf{G}_v$. Then, these expressions are used to express the received vector sample as follows:

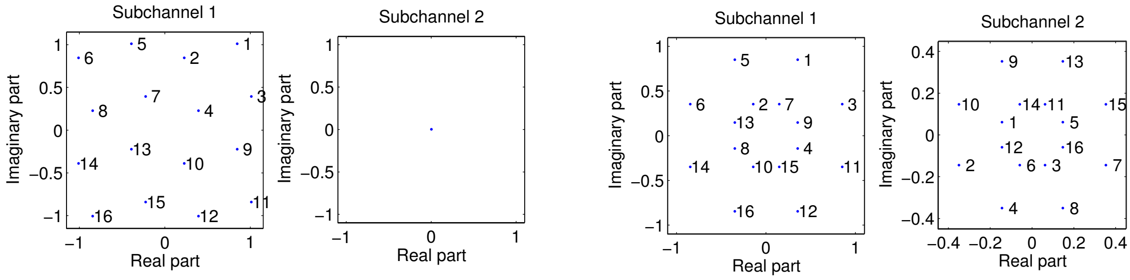
$$\mathbf{y} = \mathbf{G}_d\mathbf{H}_v\mathbf{F}_d\mathbf{s} + \mathbf{G}_d\mathbf{n}_v \tag{7}$$

where $\mathbf{H}_v = \mathbf{G}_v\mathbf{H}\mathbf{F}_v$ is the "virtual" channel, and $\mathbf{n}_v = \mathbf{G}_v\mathbf{n}$ is the "virtual" noise. The matrices $\mathbf{G}_v$ and $\mathbf{F}_v$ are chosen in order to whiten the noise ($\mathbf{R}_{n_v} = E[\mathbf{n}_v\mathbf{n}_v^*] = \mathbf{I}_{n_R}$) and diagonalize the channel ($\mathbf{H}_v$ diagonal matrix). This procedure based on the singular value decomposition (SVD) is frequently used for MIMO systems. In the case of a white noise vector $\mathbf{n}$ ($\mathbf{R}_n = \sigma_n^2\mathbf{I}_{n_R}$), the matrix $\mathbf{H}_v$ corresponds to b independent subchannels and is equal to $\mathrm{diag}(\sqrt{\lambda_1/\sigma_n^2}, \ldots, \sqrt{\lambda_b/\sigma_n^2})$ where λ_i are the b most significant eigenvalues of $\mathbf{H}\mathbf{H}^H$ with $b \leq \min(n_T, n_R)$. Furthermore, $\mathbf{G}_d$ has no effect on ML decision; this matrix is, thus, equal to the identity matrix.

In a second step, optimization of the following criterion under the constraint of $\|\mathbf{F}_d\|_F^2 = E_T$ gives $\mathbf{F}_d$:

$$\mathbf{F}_d = \arg\max_{\mathbf{F}'_d} \min_{\mathbf{s}_k,\mathbf{s}_l\in\mathcal{C}^b,\mathbf{s}_k\neq\mathbf{s}_l} \left\{\|\mathbf{H}_v\mathbf{F}'_d(\mathbf{s}_k - \mathbf{s}_l)\|^2\right\} \tag{8}$$

In the case of two independent data streams for a QPSK, the solution of (8) is found in [2]: according to the eigenvalues ratio, one gets two kinds of constellation at the receiver side depicted in Figure 4 for $\lambda_1/\lambda_2 > 10.33$ and $\lambda_1/\lambda_2 < 10.33$, respectively. The first case (a) is close to the max-SNR design used with a 16-QAM modulation (see further in the following subsection), but provides a slightly larger $d_{\min}$ due to rotation (Figure 4(a)). One should note that this solution can be easily interpreted: indeed, in the event of an obvious gain difference among the two sub-channels, only the more significant one is used. In case (b), the smaller gain is no more negligible and the precoder exploits the two subchannels in order to optimize the $d_{\min}$.



(a) case $\lambda_1/\lambda_2 > 10.33$ (b) case $\lambda_1/\lambda_2 < 10.33$

Figure 4: Two kinds of received constellations for the optimized $d_{\min}$ solution depending on the channel eigenvalues ratio. A QPSK modulation and $b = 2$ data streams are used.

Figure 4(b) shows that if two received vectors are close on one "virtual" subchannel, *e.g.* 2 and 7 on the first "virtual" subchannel, they are distant on the second one.

3.3 Maximum SNR-based precoder

Here, one calculates the SVD of the channel matrix $\mathbf{H}$ while keeping the strongest eigenvalue λ_1 and the associated vectors. The relationship with the $d_{\min}$-based precoder is then:

$$\mathbf{F} = \sqrt{E_T}\,\mathbf{F}_v \begin{pmatrix} 1 \\ \mathbf{0}_{[b-1\times 1]} \end{pmatrix}, \qquad \mathbf{G} = \begin{pmatrix} 1 & \mathbf{0}_{[1\times b-1]} \end{pmatrix} \mathbf{G}_v \tag{9}$$

This solution leads to maximize the received SNR [6]. The equivalent input-output relation is:

$$y = \sqrt{E_T \lambda_1}\, s + n \tag{10}$$

where E_T is the total average transmit power ($E[|s|^2] = 1$), and n is the AWGN with variance σ_n^2. It is worth noting that this equation is similar to the STBC equivalent scheme (2) with a different gain. The ratio r of the receiver SNR between these two schemes can be evaluated:

$$r = \frac{\text{SNR}_{\text{max-SNR}}}{\text{SNR}_{\text{OSTBC}}} = \frac{\lambda_1}{\|\mathbf{H}\|_F^2/n_T} = \frac{\lambda_1}{\sum_{i=1}^{\min(n_R,n_T)} \lambda_i/n_T} > 1 \tag{11}$$

For $n_T = 2$ (with the Alamouti code), this ratio can be expressed as $r = 2\cos^2(\gamma)$ with the variable change $\sqrt{\lambda_1} = \rho\cos\gamma$ and $\sqrt{\lambda_2} = \rho\sin\gamma$; r is totally governed by the parameter γ defined on $[0, \pi/4]$ ($\gamma = \arctan(\sqrt{\lambda_2/\lambda_1})$). In the extreme cases, *i.e.* $\gamma = 0$ ($\lambda_1 \gg \lambda_2$) and $\gamma = \pi/4$ ($\lambda_1 = \lambda_2$), the SNR gain is about 3 dB (maximum gain) and 0 dB, respectively.

3.4 d_{min} consideration

Since the minimum distance between the points of the receive constellation is directly related to the symbol error probability, in the following, the $d_{\min}$ will be evaluated for the max-SNR and the OSTBC and compared with the optimized $d_{\min}$ precoder. The expression of the minimum Euclidian distance with the QPSK modulation can be found in [2]. On the other hand, in order to always keep the same rate, the modulation used for Alamouti and Max-SNR schemes is a QAM-16. Under this assumption, the minimum distances d_{min} can be expressed by:

$$d_{Max-SNR} = \sqrt{E_T}\frac{\sqrt{10}}{5}\sqrt{\lambda_1} = \sqrt{E_T}\frac{\sqrt{10}}{5} \times \rho\cos\gamma \tag{12}$$

$$d_{Alamouti} = \sqrt{\frac{E_T}{2}}\sqrt{\lambda_1 + \lambda_2} = \sqrt{E_T}\frac{\sqrt{5}}{5} \times \rho \tag{13}$$

Figure 5 represents the normalized distance defined by $d = d_{min}/(\sqrt{E_T}\rho)$ as a function of the angle γ for the three schemes. The parameters ρ ($\rho = \sqrt{\lambda_1 + \lambda_2}$) and γ are fully characteristic of the "virtual" channel $\mathbf{H}_v$ when $b = 2$. It is worth recalling that, a small value of γ means that the first subchannel is privileged and, vice-versa, a value near 45° means two equivalent subchannels. Figure 5 also evidences that the d_{min}-based precoder has always the maximum distance. Moreover the OSTBC distance is constant and independent of the angle value. Thanks to Tx-CSI, the minimum distance is improved by the max-SNR and the d_{min}-based precoder. The $d_{\min}$-based and max-SNR precoders are close when γ is small (this corresponds to the constellation (a) in Figure 4), whereas, with constellation (b), the distance of the Max-SNR-based precoder decreases oppositely to the d_{min}-based one. In conclusion, and whatever the scheme under study, for a given parameter ρ related to the average "virtual" subchannel gain, quantification of the gain in term of normalized $d_{\min}$ is given by γ, itself related to the eigenvalues dispersion.

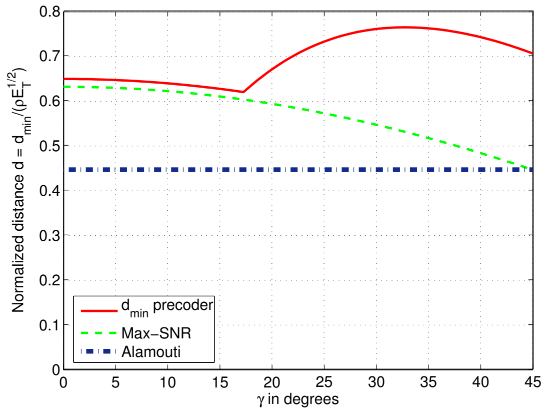


Figure 5: Normalized minimum distance as a function of γ in degrees for a given ρ

4 SIMULATION RESULTS

4.1 Polarization influence

For all simulations, we defined the SNR by E_T/σ_n^2. The parameter α determinates the polarization diversity: a high polarization diversity leads to a small value of α. In a first simulation, we only took into account the influence of α for the two extreme cases $\alpha = 0$ and $\alpha = 1$ with neither channel transmit nor receive correlations ($t = r = 0$). Figure 6 shows the bit error rate (BER) for the three systems and highlights the key role played by polarization in BER performances. The case $\alpha = 1$ is equivalent to a "true" 2×2 MIMO uncorrelated Rayleigh channel and provides systems with the maximum diversity order, $n_T \times n_R = 4$. For $\alpha = 0$, the system can be seen as two independent SISO systems at the origin of the loss in spatial diversity. When $\alpha = 1$ and RSB is high, the $d_{\min}$-based precoder gives the best performance with a gain about 0.5 dB on the max-SNR and 3.2 dB on the Alamouti scheme. For $\alpha = 0$, the $d_{\min}$-based and max-SNR precoders both loss about 6.4 dB. The Alamouti scheme is less affected by the parameter α as shown by the 5.3-dB loss.

In conclusion, α must be close to 1 to provide the best BER. The gain in cost and space offered by dual polarized antenna system made us wonder about the performances in a realistic channel. We thus simulated it with the following parameters $\alpha = 0.4$, $t = 0.5$, $r = 0.3$ given

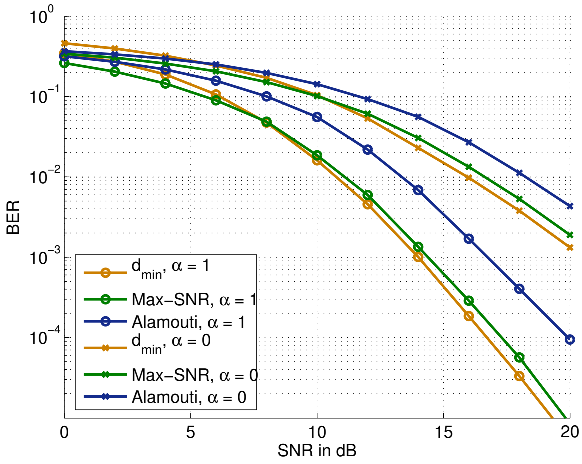


Figure 6: Influence of the polarization diversity parameter α (with the extreme cases $\alpha = 1$ and $\alpha = 0$) on BER in the case of uncorrelated Rayleigh fading

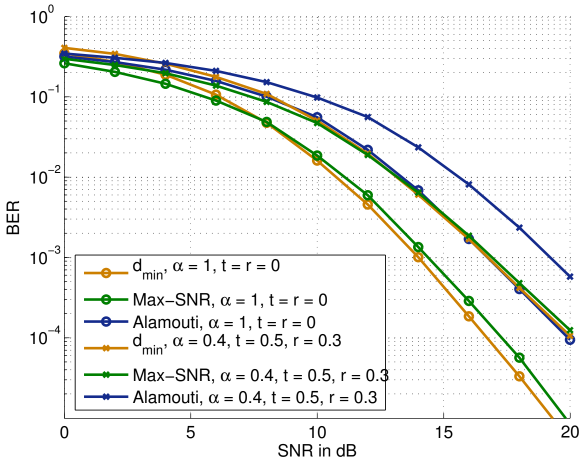


Figure 7: Comparison between an ideal MIMO uncorrelated Rayleigh channel ($\alpha = 1$, $t = r = 0$), and a realistic channel using dual polarized antennas with: $\alpha = 0.4$, $t = 0.5$ and $r = 0.3$

in [5]. As expected, Figure 7 shows that the d_{min}, Max-SNR and Alamouti schemes have a loss of gain about 3.3, 3 and 2.4 dB respectively.

4.2 Influence of channel estimation errors

Figure 8 illustrates the effects induced by addition of some errors in the channel estimation through the introduction of the matrix $\mathbf{H}_{est} = \mathbf{H} + \epsilon$ where ϵ is a matrix whose elements are i.i.d. centered complex Gaussian variables with variance $\sigma_\epsilon^2 = (4SNR)^{-1}$. It shows that the channel estimation errors have a similar influence on the three schemes and degrade performance by about 1 dB. Surprisingly, the performances of the no-TX-CSI Alamouti scheme and Tx-CSI systems show similar degradations. This results from the introduction of intersymbols interference by the channel estimation errors.

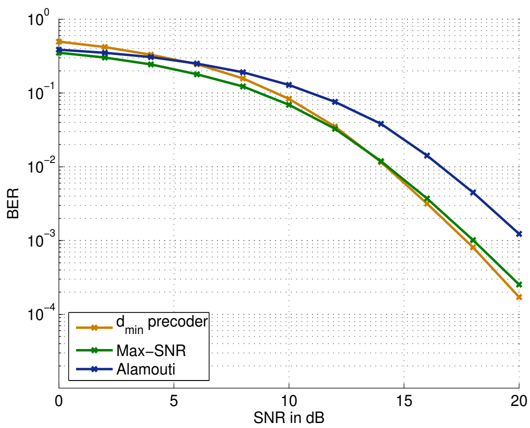


Figure 8: Simulation of BER with estimation channel errors trough a realistic Rayleigh channel ($\alpha = 0.4$, $t = 0.5$, $r = 0.3$)

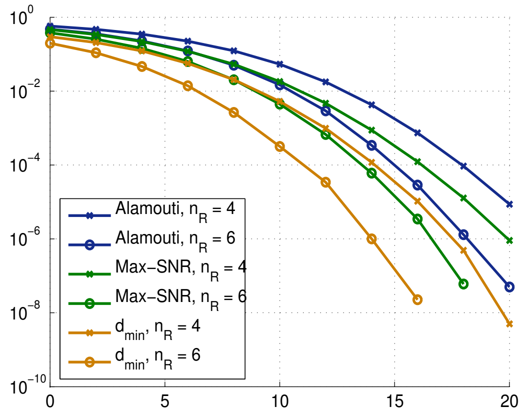


Figure 9: Simulation with two and three dual-polarized receive antennas for $\alpha = 0.4$, $t = 0.5$, $r = 0.3$

4.3 Use of several polarized receive antennas

Performances were also assessed in the case of several receive antennas by using a straightforward model assuming decorrelation of these antennas to consider the equivalent channel $2 \times n_R$ as $n_R/2$ independant channels 2×2 with the caracteristics described above (*i.e.* $\alpha = 0.4$, $t = 0.5$ and $r = 0.3$). Figure 9 illustrates the simulation data of a system with one dual-polarized transmit antenna and two or three dual-polarized receive antennas through a realistic Rayleigh fading channel. It shows the enhancement of performances about 3 dB with the $d_{\min}$ precoder for a higher number of antennas. Moreover, the Max-SNR gain on the Alamouti scheme decreases and losses about 1 dB.

5 CONCLUSION

By using a channel model to take into account correlation and polarization diversity, we simulated the performances of three schemes and observed that they were strongly affected and worsened by these parameters. Moreover, channel estimation errors had the same effect on the three schemes. In the case of a dual-polarized single transmit and receive antenna system, which is a 2x2 equivalent MIMO system, it may be worth using the Alamouti scheme due to its easy implementation and good performances. In addition, replacement of 1 dual-polarized receive antenna by 2 or 3 ones elevated the $d_{\min}$ precoder gain by more than 3 dB with respect to the Alamouti system, but highlighted the Max-SNR-precoder limit: indeed, the increase of the second subchannel gain was used by the $d_{\min}$ precoder, but neglected by the Max-SNR one. Besides, in order to use the Alamouti code because of its transmission rate equal to 1, in this study we limited the equivalent transmit antennas number to 2. More transmit antennas would impose the use of a different code with a lower rate, but a better gain for the $d_{\min}$ precoder for a same bit-rate.

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