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Limited Feedback Unitary Matrix applied to MIMO $d_{\min}$-based Precoder

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Abstract— **In spatial multiplexing systems, transmission reliability is enhanced by the full channel state information (CSI) used to design optimum linear precoders, but in practice a full CSI is often unrealistic. Indeed, a higher number of transmitters and/or receivers elevates the coefficient numbers of the estimated channel, and the precision on coefficients depends on the rate of the feedback stream. An interesting alternative is to return a limited amount of information to the transmitter; this led us to design a $d_{\min}$ precoder based on the use of *i)* a finite codebook known by the receiver and the transmitter and *ii)* feedback of only one quantized real-valued parameter. The selection criteria employed to find the optimal precoding matrix are presented. Then, the performances about BER are compared under different criteria and amounts of feedback and confronted to Alamouti code.**

I. INTRODUCTION

Thanks to rich-scattering wireless channels [1], multiple-input multiple-output (MIMO) systems have recently known an increasingly-fast development and are at the origin of significant enhancement of spectral efficiency. With no channel state information (CSI) at the transmitter side, transmission robustness can be optimized by different schemes such as Orthogonal Space Time Block Code (OSTBC) [2], [3] or [4]. When CSI is available at the transmitter side, efficient solutions like MMSE (Minimization of the Mean Square Error) [5], [6], maximum-SNR (SNR maximization at receiver side) [7], max-$d_{\min}$ precoder (maximization of the received minimal symbol vector distance) [8] allow one to improve the reliability of link by MIMO systems through transmit diversity.

A full CSI at the transmitter side is unrealistic in a real-time-varying environment because of the huge amount of feedback information to be sent back. The amount of information to be returned can be reduced by using channel statistics [9], [10], partial or quantified CSI [11]–[13].

The present study describes a quantified precoding system based on a limited-feedback unitary precoding derived from Grassmannian subspace-packing problem in order to not compromise the eigenstructure of channel matrix [14]. Indeed, the precoder max-$d_{\min}$ is based on the eigen-mode representation of the channel and the optimization of the minimum distance [8]. The max-$d_{\min}$ precoder scheme and the codebook based on the unitary precoding matrices are both used in the limited-feedback solution proposed here.

The paper is organized as follows. Section II describes the general precoded and decoded MIMO system and focuses on the optimum $d_{\min}$ precoder. In Section III, the limited feedback $d_{\min}$-based precoder is derived by using the limited feedback scheme proposed by Love and coll. Section IV deals with several criteria based on $d_{\min}$. The results of simulations carried out with an 8-bit feedback are compared with those of the classical full CSI precoder, criteria and the Alamouti code in Section V. Our conclusions are drawn in Section VI.

II. OPTIMUM $d_{\min}$PRECODER

Let us consider a (n_T, n_R) MIMO system with n_R receive and n_T transmit antennas, b independent data streams, a precoder and a decoder matrices $\mathbf{F}$ ($n_T \times b$) and $\mathbf{G}$ ($b \times n_R$), respectively, designed on assuming a perfect knowledge of channel at both sides. The basic system model is:

$$\mathbf{y} = \mathbf{GHFs} + \mathbf{Gn} \tag{1}$$

where $\mathbf{H}$ is the $n_R \times n_T$ Rayleigh flat fading channel matrix with ${}^1\mathcal{N}_c(0,1)$ i.i.d. elements, $\mathbf{s}$ is the $b \times 1$ transmitted vector symbol, and $\mathbf{n}$ is the $n_R \times 1$ additive white gaussian noise (AWGN) vector. Let us assume that $b \leq \text{rank}(\mathbf{H}) \leq \min(n_T, n_R)$ and

$$E[\mathbf{ss}^*] = \mathbf{I}_b, \ \mathbf{R}_n = E[\mathbf{nn}^*] = \sigma_n^2 \mathbf{I}_{n_R}, \ E[\mathbf{sn}^*] = \mathbf{0}. \tag{2}$$

By using the following decompositions $\mathbf{F} = \mathbf{F}_v \mathbf{F}_d$ and $\mathbf{G} = \mathbf{G}_v$, the input-output relation (1) can be rewritten as:

$$\mathbf{y} = \mathbf{H}_v \mathbf{F}_d \mathbf{s} + \mathbf{n}_v \tag{3}$$

where $\mathbf{H}_v = \mathbf{G}_v \mathbf{H} \mathbf{F}_v$ is the eigen-mode matrix channel, $\mathbf{n}_v = \mathbf{G}_v \mathbf{n}$ is the additive noise vector on the channel eigen-mode with the covariance matrix $\mathbf{R}_{n_v} = E[\mathbf{n}_v \mathbf{n}_v^*] = \sigma_n^2 \mathbf{I}_b$, the unitary matrices, $\mathbf{G}_v$ and $\mathbf{F}_v$, are chosen so as to diagonalize the channel and to reduce dimension to b. The matrix, $\mathbf{F}_d$, results from optimization under a specific criterion.

To calculate $\mathbf{F}_d$ we used the max-$d_{\min}$ strategy. The minimum Euclidean distance is defined by:

$$d_{\min}(\mathbf{F}_d) = \min_{\mathbf{s}_k, \mathbf{s}_l \in \mathcal{C}^b, \mathbf{s}_k \neq \mathbf{s}_l} \|\mathbf{H}_v \mathbf{F}_d (\mathbf{s}_k - \mathbf{s}_l)\|. \tag{4}$$

The max-$d_{\min}$ precoder is the solution of:

$$\mathbf{F}_d = \arg\max_{\mathbf{F}'_d} d_{\min}(\mathbf{F}'_d) \tag{5}$$

[1] $\mathcal{N}_c(0,1)$ is the zero-mean and unit-variance complex normal distribution, $\mathcal{C}$ is the symbol constellation, $\mathbf{I}_n$ is the identity matrix $n \times n$, $(.)^*$ is the transpose conjugate and $\|.\|_F$ is the Frobenius norm.

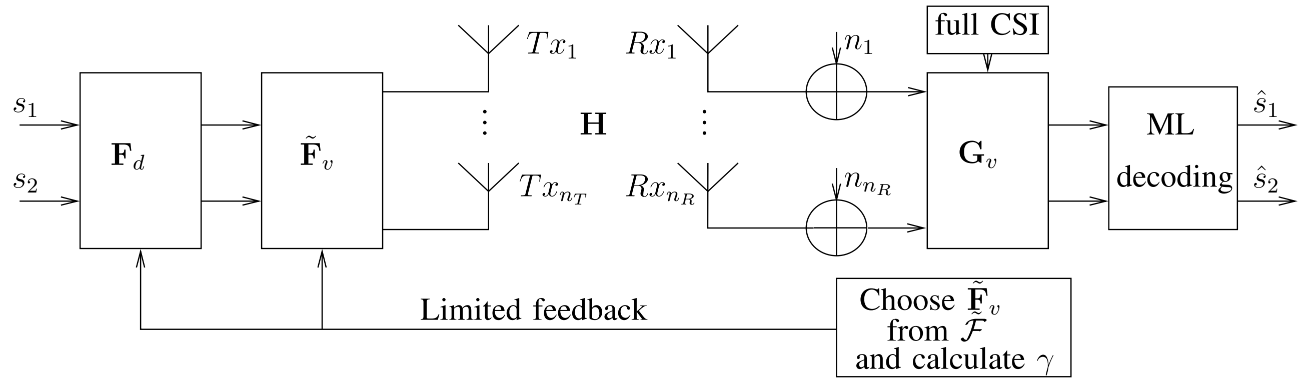

Fig. 1. Block diagram of the $d_{\min}$ based precoder with limited feedback system

under the power constraint $\|\mathbf{F}_d\|_F^2 = E_T$. The solution of (5) is difficult, and a very exploitable solution was given in [8] for two independent data streams, $b = 2$ and a 4-QAM. In this case, the matrix $\mathbf{H}_v = \mathrm{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2})$ corresponds to the two independent subchannels ($\lambda_1 \geq \lambda_2$ are the two principal eigenvalues of $\mathbf{HH}^*$). This solution of $\mathbf{F}_d$ is calculated by:

for $0 \leq \gamma \leq \gamma_0$,

$$\mathbf{F}_d = \mathbf{F}_{r1} = \sqrt{E_T}\begin{pmatrix} \sqrt{\frac{3+\sqrt{3}}{6}} & \sqrt{\frac{3-\sqrt{3}}{6}}\, e^{i\frac{\pi}{12}} \\ 0 & 0 \end{pmatrix} \quad (6)$$

for $\gamma_0 \leq \gamma \leq \pi/4$,

$$\mathbf{F}_d = \mathbf{F}_{octa} = \sqrt{\frac{E_T}{2}}\begin{pmatrix} \cos\psi & 0 \\ 0 & \sin\psi \end{pmatrix}\begin{pmatrix} 1 & e^{i\frac{\pi}{4}} \\ -1 & e^{i\frac{\pi}{4}} \end{pmatrix} \quad (7)$$

where $\gamma = \arctan\sqrt{\frac{\lambda_2}{\lambda_1}}$ is an angle linked to the eigenvalues ratio, ψ is related to the eigen-mode power allocation, and $\gamma_0 \simeq 17.28°$ is a constant threshold; moreover, ψ depends on γ with $\psi = \arctan\frac{\sqrt{2}-1}{\tan\gamma}$. Eqs. (6) and (7) can be directly computed to design the max-$d_{\min}$ precoder for a given channel matrix. One should note that the d_{min} precoder requires only one parameter γ to evaluate $\mathbf{F}_{r1}$ or $\mathbf{F}_{octa}$.

III. Limited Feedback $d_{\min}$-based Precoder

The Grassmannian theory [15] was employed by Love *et al.* [14] to compute codebooks, $\tilde{\mathcal{F}}$ with unitary matrix $\tilde{\mathbf{F}}_v \in \tilde{\mathcal{F}}$. On condition to assume a Rayleigh MIMO channel, this theory permits one to minimize the average distortion by meeting:

$$E_{\mathbf{H}}\left[\min_{\tilde{\mathbf{F}}_v \in \tilde{\mathcal{F}}}\left(\|\mathbf{HF}_v\|_F^2 - \|\mathbf{H}\tilde{\mathbf{F}}_v\|_F^2\right)\right]. \quad (8)$$

It is worth recalling briefly how these authors used a practical codebook, $\tilde{\mathcal{F}}$, leading to a family of N matrices. It is defined as:

$$\tilde{\mathcal{F}} = \left\{\mathbf{F}_{DFT}, \Theta\mathbf{F}_{DFT}, \ldots, \Theta^{N-1}\mathbf{F}_{DFT}\right\} \quad (9)$$

where $\mathbf{F}_{DFT}$ is an $n_T \times b$ matrix with entry (k, l) equal to $\left(1/\sqrt{n_T}\right) e^{-j2\pi kl/n_T}$ and Θ is a diagonal matrix given by

$$\Theta = \begin{bmatrix} e^{-j2\pi u_1/n_T} & 0 & \ldots & 0 \\ 0 & e^{-j2\pi u_2/n_T} & \ldots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \ldots & e^{-j2\pi u_{n_T}/n_T} \end{bmatrix} \quad (10)$$

where $0 \leq u_1, \ldots, u_{n_T} \leq N-1$.

The coefficients $u_1, \ldots, u_{n_T}$ must be chosen for maximizing the minimum distance[2] between the reference matrix, $\mathbf{F}_{DFT}$, and the matrices, $\Theta^l\mathbf{F}_{DFT}$, as follows:

$$\mathbf{u} = \arg\max_{\mathcal{Z}} \min_{1\leq l\leq N-1} d\left(\mathbf{F}_{DFT}, \Theta^l\mathbf{F}_{DFT}\right). \quad (11)$$

where $\mathbf{u} = [u_1, u_2 \ldots, u_{n_T}]^T$ and the set $\mathcal{Z} = \{\mathbf{u} \in \mathbb{Z}^{n_T} | \forall k, 0 \leq u_k \leq N-1\}$.

Since $\mathbf{G}_v$ is known at the receiver side, as illustrated in Fig. 1, the feedback of the index matrix from $\tilde{\mathcal{F}}$ and that of the coefficient γ are key-parameters.

One should note that, to store the codebook $\tilde{\mathcal{F}}$ at the transmitter/receiver, only the coefficients $u_1, u_2 \ldots, u_{n_T}$ are needed; this codebook corresponds to $n_T \log_2 N$ bits. The matrix, $\tilde{\mathbf{F}}_v$, can be computed by using the binary codeword index l with $\Theta^{l-1}\mathbf{F}_{DFT}$.

IV. $d_{\min}$Global System

The global system under study associates the max-$d_{\min}$ precoder and the limited feedback unitary precoder: Fig. 1 represents the block diagram. The symbol vector $\mathbf{s} = [s_1\ s_2]^T$ is precoded by meeting the condition of maximization of $d_{\min}$ distance by the matrix $\mathbf{F}_d$ ($\mathbf{F}_{r1}$ or $\mathbf{F}_{octa}$); the only coefficient needed for the matrix calculation is γ. To guarantee full diversity order, the matrices $\tilde{\mathbf{F}}_v$ and $\mathbf{G}_v$ are respectively used at the transmitter and receiver; $\tilde{\mathbf{F}}_v$ is chosen within the codebook $\tilde{\mathcal{F}}$ by returning the index estimated at the receiver side. To be selected, the unitary matrix, $\tilde{\mathbf{F}}_v$, needs to meet at best the chosen criterion.

At first, to choose the unitary matrix, $\tilde{\mathbf{F}}_v$, we selected four criteria denoted as follows:

[2]The distance calculated here is the *chordal distance* defined by $d(\mathbf{A}, \mathbf{B}) = \frac{1}{\sqrt{2}}\|\mathbf{AA}^* - \mathbf{BB}^*\|_F$

1) Name: $\min \mathbf{H}_v$

$$\tilde{\mathbf{F}}_v = \arg\min_{\tilde{\mathbf{F}}'_v \in \tilde{\mathcal{F}}} \left\| \mathbf{H}_v - \tilde{\mathbf{H}}_v \right\|_F \qquad \text{with} \quad \tilde{\mathbf{H}}_v = \mathbf{G}_v \mathbf{H} \tilde{\mathbf{F}}'_v. \tag{12}$$

The max-$d_{\min}$ precoder uses $\tilde{\mathbf{H}}_v$ to optimize $d_{\min}$.

2) Name: max-$d_{\min}(\mathbf{F}_v)$

$$\tilde{\mathbf{F}}_v = \arg\max_{\tilde{\mathbf{F}}'_v \in \tilde{\mathcal{F}}} \left\{ \min_{\varepsilon_k} \left\| \tilde{\mathbf{H}}_v \mathbf{F}_d \varepsilon_k \right\| \right\}, \tag{13}$$

where $\varepsilon_k = \mathbf{s}_m - \mathbf{s}_l$ $(m \neq l)$ is the set of differences between the possible transmitted symbol vectors. One should note that, as some vectors are collinear, for a 4-QAM the set of 240 elements can be reduced to 14. The matrix, $\mathbf{F}_d$, is calculated with (6) and (7), and γ is given by $\gamma = \left| \arctan \sqrt{\frac{\tilde{\mathbf{H}}_v(2,2)}{\tilde{\mathbf{H}}_v(1,1)}} \right|$. Computation of this criterion requires to calculate $14 \times N$ distances.

3) Name: max-$d_{\min}(\mathbf{F}_v,\gamma)$

$$(\tilde{\mathbf{F}}_v, \gamma) = \arg\max_{\tilde{\mathbf{F}}'_v \in \tilde{\mathcal{F}}, \gamma' \in [0,\pi/4]} \left\{ \min_{\varepsilon_k} \left\| \tilde{\mathbf{H}}_v \mathbf{F}_d \varepsilon_k \right\| \right\}. \tag{14}$$

This criterion counterbalances the average distortion by γ and $\tilde{\mathbf{F}}_v$, which are both optimized. It is worth underlining that the search over γ has no detrimental effect on the duration of computation; by using a standard optimization function, a satisfying solution is found with only few iterations.

4) Name: $\text{unitary} - \text{max-}d_{\min}$

$$\tilde{\mathbf{F}}_v = \arg\max_{\tilde{\mathbf{F}}'_v \in \tilde{\mathcal{F}}} \left\{ \min_{\varepsilon_k} \left\| \tilde{\mathbf{H}}_v \varepsilon_k \right\| \right\}. \tag{15}$$

This criterion was proposed in [14]; a single unitary matrix $\tilde{\mathbf{F}}_v$ is required to optimize $d_{\min}$. As a result, the max-$d_{\min}$ precoder, described in Section II is of no use here.

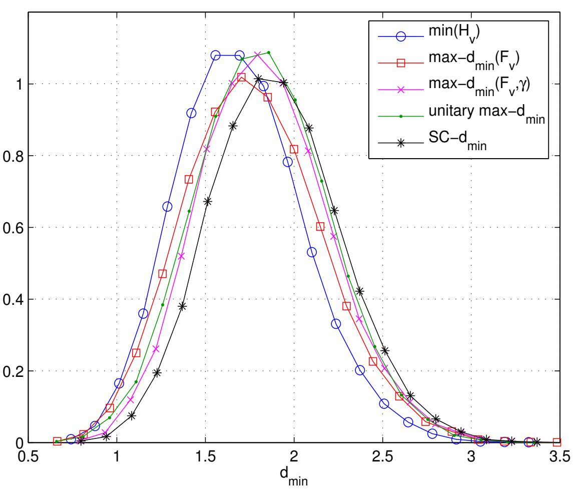

Fig. 2. $d_{\min}$ normalized histogram for the different criteria

TABLE I

PROBABILITY OF $d_{\min}$ FOR max-$d_{\min}(\mathbf{F}_v,\gamma)$, $\text{unitary} - \text{max-}d_{\min}$ AND SC-$d_{\min}$ CRITERIA

	$p_{d_{\min}}(x<1.2)$	$p_{d_{\min}}(x>2.5)$
max-$d_{\min}(\mathbf{F}_v,\gamma)$	0.0274	0.0376
$\text{unitary} - \text{max-}d_{\min}$	0.0369	0.0423
SC-$d_{\min}$	0.0218	0.0467

Fig. 2 presents the $d_{\min}$ probability density function (p.d.f.: normalized histograms) calculated with these criteria where $n_T = 2$ and $n_R = 4$ with 30 000 Rayleigh channel matrices $\mathbf{H}$. It shows that the weakest $d_{\min}$ is given by the $\min \mathbf{H}_v$ criterion, and then by the max-$d_{\min}(\mathbf{F}_v)$ one. Concerning the max-$d_{\min}(\mathbf{F}_v,\gamma)$ and $\text{unitary} - \text{max-}d_{\min}$ criteria: for $d_{\min}$, the former is better; on the other hand, for high $d_{\min}$, the $\text{unitary} - \text{max-}d_{\min}$ criterion works better. These considerations led us to design, in a second step, a 5^{th} criterion, denoted hereafter limited feedback switch $d_{\min}$ (SC-$d_{\min}$). To obtain the best $d_{\min}$, SC-$d_{\min}$ switches from max-$d_{\min}(\mathbf{F}_v,\gamma)$ to $\text{unitary} - \text{max-}d_{\min}$, and conversely. This p.d.f. is illustrated in Fig. 2, by the curve with '*'. Table I details the probability of $d_{\min}$ for max-$d_{\min}(\mathbf{F}_v,\gamma)$, $\text{unitary} - \text{max-}d_{\min}$ and SC-$d_{\min}$ criteria when $p_{d_{\min}}(x<1.2)$ and $p_{d_{\min}}(x>2.5)$.

V. SIMULATION

A. BER Performances

For the simulation of MIMO system performance, we set the number of antennas to $n_T = 2$ and $n_R = 4$, 30 000 channel matrices $\mathbf{H}$ and 1 000 symbol vectors $\mathbf{s}$ were generated per $\mathbf{H}$, the codebook length was equal to $N = 2^4 = 16$, *i.e.* 4 bits, and the exact value of the angle γ was first fed back, but its quantization will be treated later. Fig. 3 compares the performances between full CSI and limited feedback criteria: $\min \mathbf{H}_v$, max-$d_{\min}(\mathbf{F}_v)$, max-$d_{\min}(\mathbf{F}_v,\gamma)$, $\text{unitary} - \text{max-}d_{\min}$ and SC-$d_{\min}$. It shows that the curves are close in low SNR; on the other hand, when the SNR is increasing, the performances of the max-$d_{\min}$ criteria $\min \mathbf{H}_v$ and max-$d_{\min}(\mathbf{F}_v)$ are degraded. At a BER equal to 10^{-5}, the loss is up to 2 dB for $\min \mathbf{H}_v$ and up to 1.3 dB for max-$d_{\min}(\mathbf{F}_v)$when compared to max-$d_{\min}$ full CSI. For max-$d_{\min}(\mathbf{F}_v,\gamma)$, $\text{unitary} - \text{max-}d_{\min}$ and SC-$d_{\min}$ the performance degradation is less than 1 dB. SC-$d_{\min}$ is close to full CSI (<0.3 dB). So, with only 4 bits of feedback information and an exact value for γ, the max-$d_{\min}(\mathbf{F}_v,\gamma)$, $\text{unitary} - \text{max-}d_{\min}$ and SC-$d_{\min}$ are always competitive compared to the system with full CSI.

To explain the good performances of criterion SC-$d_{\min}$, Fig. 6 presents the p.d.f. of the condition number C.N.:

- the normalized histogram of C.N. when $\text{unitary} - \text{max-}d_{\min}$ is chosen

$$f_{CN/\text{unitary} - \text{max-}d_{\min}}, \tag{16}$$

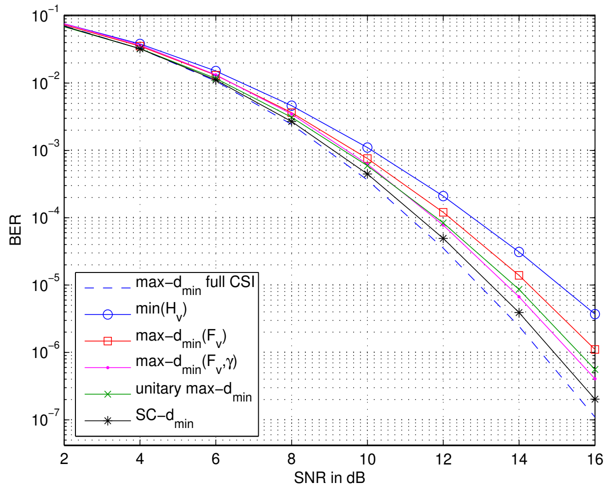

Fig. 3. BER comparison between max-$d_{\min}$ with full CSI and limited feedback criteria

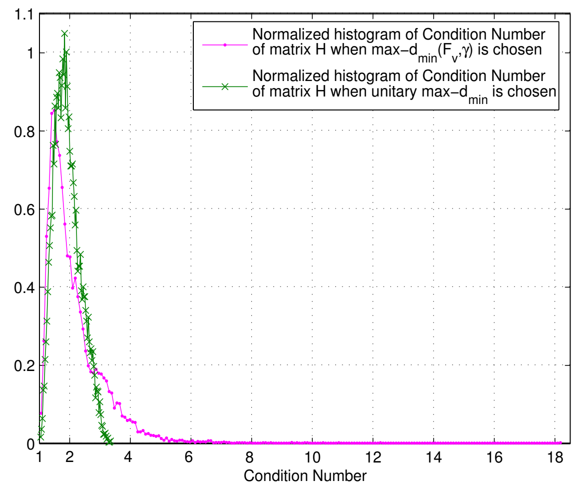

Fig. 4. Condition number normalized histogram of the two criteria used for SC-$d_{\min}$

- the normalized histogram of C.N. when max-$d_{\min}(\mathbf{F}_v,\gamma)$ is chosen

$$f_{CN/\text{max-}d_{\min}(\mathbf{F}_v,\gamma)}. \tag{17}$$

The C.N. is the ratio between the largest and the smallest eigenvalues of the channel matrix, and is greater then 1 ($\lambda_1 > \lambda_2$). The condition number p.d.f$_s$. are calculated from the eigenvalues λ_1 and λ_2 of $\mathbf{H}_v$ whenever either max-$d_{\min}(\mathbf{F}_v,\gamma)$ (dot curve) or unitary − max-$d_{\min}$ (cross curve) are chosen. The C.N. p.d.f. of criterion max-$d_{\min}(\mathbf{F}_v,\gamma)$ shows that this criterion is used when the $\mathbf{H}$ matrices is ill-conditioned (C.N.>3.4). On the other hand, unitary − max-$d_{\min}$ is used only for well-conditioned $\mathbf{H}$ (C.N.<3.4). An ill-conditioned $\mathbf{H}$ matrix degrades the performances when the power allocation is uniform (criterion unitary − max-$d_{\min}$). It appeared that, in this configuration, with 30 000 $\mathbf{H}$ matrices, the max-$d_{\min}(\mathbf{F}_v,\gamma)$ criterion was selected more frequently than the unitary − max-$d_{\min}$ criterion (54% against 46%); this difference corresponds approximatively to the tail of the dot curve, when the max-$d_{\min}(\mathbf{F}_v,\gamma)$ criterion is used to enhance $d_{\min}$. In the range 1<C.N.<3.4, max-$d_{\min}(\mathbf{F}_v,\gamma)$ criterion is used at 49.2%. Then, Fig. 3 shows that the max-$d_{\min}(\mathbf{F}_v,\gamma)$ criterion is better better than the unitary − max-$d_{\min}$ one at high SNR because of different power allocation to sub-streams to compensate for ill-conditioned matrices $\mathbf{H}$.

B. Quantization of γ

Fig. 3 presents the performances with perfect real value γ. Let us, now, consider the quantization of the real value γ and analyze the behavior of return information. Fig. 5 depicts the length of the index codebook equal to n_1 bits and the real value $\gamma \in [0,\pi/4]$ quantized by 2^{n_2}. The quantization is realized by uniformly dividing the range by 2^{n_2}. One should note that, when SC-$d_{\min}$ is used, $00\ldots0$ sequence is sent to the transmitter as γ value, and then the transmitter chooses the unitary − max-$d_{\min}$ criterion; otherwise, the quantized γ is sent and the transmitter selects the max-$d_{\min}(\mathbf{F}_v,\gamma)$ criterion with the quantized angle.

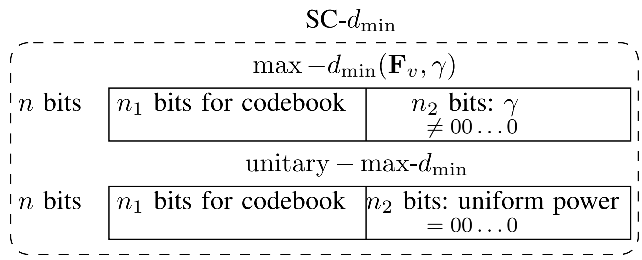

Fig. 5. Number of bits used to fed back for SC-$d_{\min}$ criterion

To evidence the impact of γ quantization on performances, simulations were carried out with a bit number n_2 equal to 2, 3 and 4 and $n_1 = 4$ for the codebook index. Fig. 6 shows the criterion max-$d_{\min}(\mathbf{F}_v,\gamma)$ for different n_2. According to the results, γ is affected by bit quantization, and BER is degraded. But on condition to have only 4 bits, the performances are decreased by about 0.15 dB compared to the exact transmitted value.

C. Performance comparisons between SC-$d_{\min}$, full CSI max-$d_{\min}$ and OSTBC

Finally, to assess the benefits of our totally limited feedback switch precoder with respect to the performances produced by the full CSI max-$d_{\min}$ precoder, the SC-$d_{\min}$ precoder and the Alamouti code we performed simulations under the following conditions: $n_T = 2$ and $n_R = 4$, 30 000 $\mathbf{H}$, 1 500 symbols per $\mathbf{H}$ and $n_1 = 4$ bits and $n_2 = 4$ bits. To meet the bit rate, two 4-QAM were used for both the full CSI max-$d_{\min}$ precoder and the SC-$d_{\min}$ precoder against two 16-QAM for the Alamouti code. Fig. 7 shows undoubtedly that the SC-$d_{\min}$ precoder behaves nearly alike the full CSI with a loss less than 0.4 dB

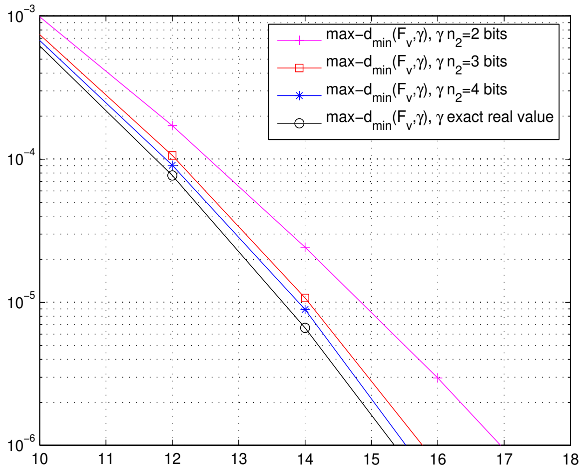

Fig. 6. Impact of γ quantization on BER for $\max\text{-}d_{\min}(\mathbf{F}_v, \gamma)$criterion

for a BER equal to 10^{-5}. Compared to the Alamouti code that uses no precoder, the gain obtained by adding only 8 bits of feeback for SC-$d_{\min}$ is equal to 3 dB.

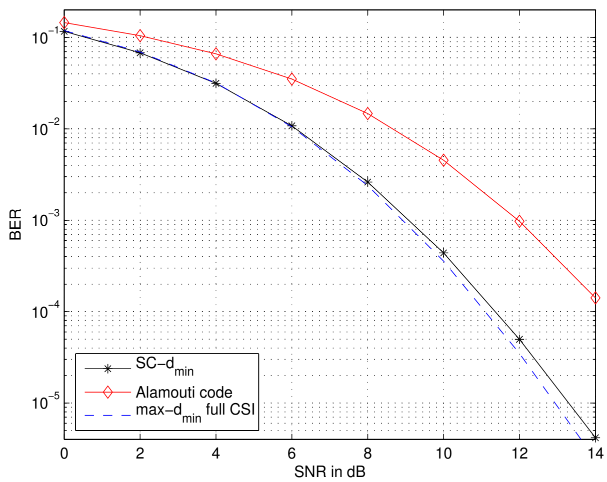

Fig. 7. BER comparison between $\max\text{-}d_{\min}$ with full CSI, SC-$d_{\min}$ and and Alamouti code, MIMO (2,4)

VI. Conclusion

This investigation were focused on the limited feedback information for the $\max\text{-}d_{\min}$ precoder was studied. Indeed, many precoders need full CSI at the transmitter side to optimize the power allocation. Then, after the description of the $\max\text{-}d_{\min}$ precoder, we designed a limited feedback $d_{\min}$-based precoder using a practical codebook to guarantee the full diversity order. To optimize the $d_{\min}$ distance, we proposed four criteria for the selection of the best codebook: $\min \mathbf{H}_v$, $\max\text{-}d_{\min}(\mathbf{F}_v)$, $\max\text{-}d_{\min}(\mathbf{F}_v, \gamma)$ and $\text{unitary} - \max\text{-}d_{\min}$. Analysis of results drove us to design a fifth criterion, SC-$d_{\min}$, to switch from a joint optimization of the $\max\text{-}d_{\min}$ precoder and codebook to a unitary precoder and conversely. This new limited feedback precoder provided very good performances close to those of the full CSI $\max\text{-}d_{\min}$ precoder with a loss of only 0.4 dB in a (2,4) MIMO system. Compared to the Alamouti code (no CSI at the transmitter side), the gain produced by the SC-$d_{\min}$ was equal to 3 dB, with only an 8-bit return to the transmitter.

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