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Parallel Blind Multiuser Synchronization and Sequences Estimation in Multirate CDMA Transmissions

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***Abstract*— In this paper, the blind synchronization approach based on the FROBENIUS Square Norm Behaviour (FSNB) proposed in [1]–[3] is extended to the case of several users in multirate CDMA transmissions. It consists in estimating the FSNB criterion maxima (synchronization peaks) positions, and it allows one to estimate desynchronization and transmissions delay times. We prove that, the extented-FSNB criterion is a powerful tool for blind synchronization. We also detail the synchronization peaks masking which degrades its performances especially at very low SNRs, and show how to proceed in this case. Simulation results illustrate sequences identification and symbols recovering, after synchronizing using the extended-FSNB.**

I. INTRODUCTION

Many blind schemes and algorithms have been devised to either improve the performance or reduce the complexity of a CDMA receiver in a multiuser context. Some prior knowledge of user, e.g. the signature waveform [4], the processing gain, the chip rate [5], is always assumed, but its nature depends on the technique employed. Here, we propose a blind multiuser synchronization schemes with no prior knowledge about the transmitter. Typically, it is the case for blind signals interception in the military field or for spectrum surveillance.

In this article, we extend the FROBENIUS Square Norm Behaviour (FSNB)-based criterion previously proposed in [2], [3] to the multiuser case. Hence, this report is organized as follows: Section II will introduce the signal model and assumptions made, while Section III will present our proposed approach. Section IV will deal with the theoretical analysis of the FSNB-based criterion. The simulations results will be detailed in Section V, and our conclusions will be drawn in Section VI.

II. SIGNAL MODELING AND ASSUMPTIONS

The multirate CDMA uplink, where the data rate is tied to the corresponding sequence length (Variable Spreading Length technique), is considered throughout this paper. By denoting $\{\mathcal{R}_0, \mathcal{R}_1, \cdots, \mathcal{R}_{S-1}\}$ a set of S available data rate ranked in ascending order, N_u^i the number of active users transmitting at $\mathcal{R}_i$ and N_u the total number of users such that $\sum_{i=0}^{S-1} N_u^i = N_u$, the complex received signal can be expressed as:

$$y(t) = \sum_{i=0}^{S-1} \sum_{n=0}^{N_u^i-1} \sum_{k=-\infty}^{+\infty} a_{n,i}(k) h_{n,i}(t - kT_{s_i} - T_{d_{n,i}}) + b(t) \quad (1)$$

where $h_{n,i}(t) = \sum_{k=0}^{L_i-1} c_{n,i}(k) p_i(t - kT_c)$. In (1), the subscript $(\cdot)_{n,i}$ refers to the n^{th} user transmitting at $\mathcal{R}_i$, denoted throughout this report as the $(n,i)^{th}$ user. Accordingly:

- $a_{n,i}(k)$ are the baseband symbols of variance $\sigma^2_{a_{n,i}}$ for the $(n,i)^{th}$ user, whereas $p_i(t)$ is the convolution of the transmission filter, channel filter (which takes into account channel echoes, fading, multipaths and jammers) and receiver filter for each rate.
- The term $h_{n,i}(t)$, defined in $[0 \quad T_{s_i}[$, is a virtual filter corresponding to the convolution of all filters of the transmission chain with the spreading sequence $\{c_{n,i}(k)\}_{k=0\cdots L_i-1}$, where L_i is the spreading factor for the $(n,i)^{th}$ user.
- Because of the VSL technique, the symbol period T_{s_i} for the users transmitting at the rate R_i is tied to the common chip period T_c: $T_{s_i} = L_i T_c$, and $s_{n,i}$ stands for the $(n,i)^{th}$ signal.
- The term $T_{d_{n,i}}$ is the corresponding transmission delay for the $(n,i)^{th}$ user; it is assumed to satisfy: $0 \leqslant T_{d_{n,i}} < T_{s_i}$ and to remain constant during the observation.
- $b(t)$ is a centered white Gaussian noise of variance σ_b^2.
- Signals are assumed to be independent, centered, noise-uncorrelated and received with the same power: $\sigma^2_{s_{n,i}} = \sigma^2_{s_{0,0}}$, for all (n,i).
- Finally, the SNR (in dB) at the detector intput is negative (signal hidden in the noise).

III. PROPOSED APPROACH

The blind multiuser detection scheme in multirate CDMA systems recently proposed in [6] allows one to estimate different symbol periods T_{s_i}, thus, various data rates. Hence, the

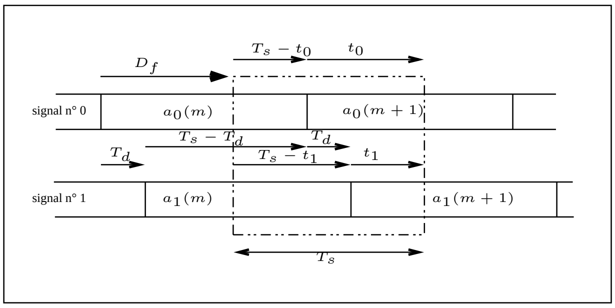

Figure 1. Relative positions of signals and an analysis window before the synchronization process, $N_u^i = 2$ in uplink.

synchronization process is performed in parallel in each group i of users transmitting at the same data rate, as described as follows.

Firstly, the intercepted signal is sampled and divided into N non-overlapping temporal windows of duration $T_F = T_{s_i} = MT_e$, $M \in \mathbb{N}^\star$, like in [2], [3]; where T_e is the sampling period, T_{s_i} is the symbol period estimated in [6]. Consequently, each window contains M samples. Contrary to [7], the sampling and chip periods are not equals, and the number of samples per window is not equal to sequences length, since those parameters are not known. Then, in each group i, and in parallel, the correlation matrix of the sampled-received signal is computed.

A. Correlation matrix computation

For more clearness, the subscript $(\cdot)_{n,i}$ will be denoted $(\cdot)_n$ and T_{s_i} will be noted T_s whithin each group i, since all active users are transmitting at the same data rate. Thus, (1) becomes:

$$\begin{aligned} y(t) &= \sum_{n=0}^{N_u^i-1} \sum_{k=-\infty}^{+\infty} a_n(k) h_n(t - kT_s - T_{d_n}) + b(t) \\ &= s(t) + b(t) \end{aligned} \tag{2}$$

where $s(t)$ stands for the noise-unaffected received signal. By denoting $\mathbf{y}_e(t)$ the vector containing the sampled received signal $s(t)$, we get:

$$\mathbf{y}_e(t) = [s(t), s(t+T_e), \cdots, s(t+(M-1)T_e)] \tag{3}$$

Then we compute the $(M \times N)$-matrix $\mathbf{Y}_e$ which N columns contain M signal samples, as showed on (4).

$$\mathbf{Y}_e = \begin{bmatrix} s(t) & \cdots & s(t+(N-1)T_s) \\ \vdots & \ldots & \vdots \\ s(t+T_s-T_e) & \cdots & s(t+NT_s-T_e) \end{bmatrix} \tag{4}$$

For a better understanding of the sequel, let us analyse Fig. 1, where D_f stands for an analysis window shifts (here, from left towards right), T_d for the transmission time delay of signal n^o 1, t_0 and t_1 represent the temporal shift between the analysis window and the beginning of a whole symbol of each signal, respectively. It clearly appears that, according to analysis window shifts D_f, the shifts t_0 and t_1 values will change, hence, the matrix $\mathbf{Y}_e$ elements will also change.

Since the vectors $h_n(t)$ are defined in $[0 \quad T_s[$, theoretical analysis allowed us to prove that the matrix $\mathbf{Y}_e$ can be expressed as:

$$\mathbf{Y}_e = \sum_{n=0}^{N_u^i-1} \left(\mathbf{h}_n^0 + \mathbf{h}_n^{-1}\right) \mathbf{a}_n^T \tag{5}$$

where the vector $\mathbf{a}_n^T = [\cdots, a_n(m) \cdots]$ contains all the symbols of n^{th} user, and vectors $\mathbf{h}_0^n$ and $\mathbf{h}_n^{-1}$ are defined, for each interfering user whithin the group i, as follows :

- the vector $\mathbf{h}_n^{-1}$ contains the end of the corresponding spreading waveform during $T_s - t_n$, followed by zeros during t_n;
- the vector $\mathbf{h}_n^0$ contains zeros during t_n, followed by the beginning of the corresponding spreading waveform during $T_s - t_n$.

Vectors $\mathbf{h}_n^0$ and $\mathbf{h}_n^{-1}$ allow one to take into account the temporal shifts t_n between an analysis window and the beginning of a whole symbol of the n^{th} interfering signal in the correlation matrix expression. In fact, by noting $\mathbf{b}$ the noise $(M \times N)$-matrix, the received noise-affected $(M \times M)$ correlation matrix $\mathbf{R}$ is given by:

$$\mathbf{R} = (\mathbf{Y}_e + \mathbf{b})(\mathbf{Y}_e + \mathbf{b})^* \tag{6}$$

where $\{(\cdot)^*\}$ denotes the conjugate transpose of $(\cdot)$. Thus, like in [2], [3], with the assumptions of independent, centered and noise-uncorrelated signals, and since the signals are assumed to be received with the same power, the simplified correlation matrix can be expressed as:

$$\mathbf{R} = \sigma_b^2 \left\{ \beta \sum_{n=0}^{N_u^i-1} \left\{ (1-\alpha_n)\mathbf{v}_n^0 (\mathbf{v}_n^0)^* + \alpha_n \mathbf{v}_n^{-1} (\mathbf{v}_n^{-1})^* \right\} + \mathbf{I} \right\} \tag{7}$$

where $\beta = \rho \frac{T_s}{T_e}$, with ρ being the signal to noise and interference ratio (SNIR) compared to one of the N_u^i users, $\mathbf{v}_n^0$ and $\mathbf{v}_n^{-1}$ are normalized vectors of $\mathbf{h}_n^0$ and $\mathbf{h}_n^{-1}$, $\alpha_n = \frac{t_n}{T_s}$, and $\mathbf{I}$ is the identity matrix.

Equation (7) shows that for any n, couples $\left\{\mathbf{v}_n^0, \mathbf{v}_{n+1}^0\right\}$ and $\left\{\mathbf{v}_n^{-1}, \mathbf{v}_{n+1}^{-1}\right\}$ span two orthogonal subspaces of dimension 2. Consequently, each subspace generates two eigenvectors and eigenvalues. An Eigenvalue Decomposition (EVD) highlights $2N_u^i$ eigenvalues associated to the signal space and the $M - 2N_u^i$ ones associated to the noise space (all are assumed to be equal on average to the noise power), as described follows.

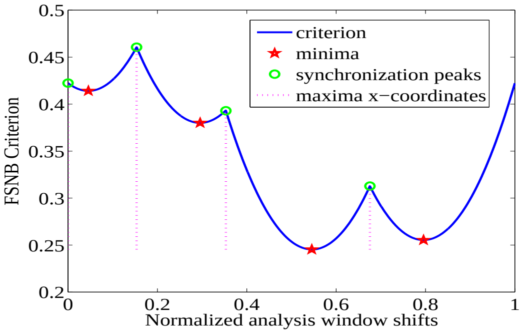

Figure 2. Theoretical FSNB-based criterion F, $N_u^i = 4$

B. FSNB-based criterion definition

Since sequences are assumed almost uncorrelated, eigenvalues in an unspecified order are:

$$\begin{cases} \lambda_n^0 = \sigma_b^2 \left\{\beta\left(1-\alpha_n\right)+1\right\}, \quad n=0,\cdots,N_u^i-1 \\ \lambda_n^{-1} = \sigma_b^2 \left\{\beta\alpha_n+1\right\}, \quad n=0,\cdots,N_u^i-1 \\ \lambda_n = \sigma_b^2, \quad n=2N_u^i,\cdots,M-1 \end{cases} \tag{8}$$

Then, we prove that the eigenvalues sum not depends on both transmission delays and analysis windows shifts: it is constant. Then, the FROBENIUS square norm of (7), defined as the sum of the square eigenvalues (8) is computed. In addition, we show that this square norm has a constant part and a variable part. Hence, we define the blind synchronization criterion, denoted F, based on the correlation FROBENIUS square norm behaviour (FSNB), as being the variable part of the FROBENIUS square norm of $\mathbf{R}$:

$$F(\alpha_0,\cdots,\alpha_n) = 1 + \sum_{n=0}^{N_u^i-1}\left(\alpha_n^2-\alpha_n\right) \tag{9}$$

The FSNB synchronization process consists maximizing the FROBENIUS square norm of $\mathbf{R}$, which is equivalent in determinating the criterion F maxima (synchronization peaks). However, F depends of several variables $(\alpha_0,\cdots,\alpha_n)$. Hence, in the following section, (9) will be rewritten in order to make it more simple to study.

IV. THEORETICAL ANALYSIS

For a better comprehension, let us analyze again Fig.1. When the analysis window shift is null, the signal $n^0 0$ is synchronized, since the analysis window contains a whole symbol of the corresponding signal. When the analysis window shift is equal to the transmission delay between the two users, the signal $n^0 1$ is synchronized, and finally, when the analysis window shift is equal to the symbol period, whe refind the initial configuration, i.e. the analysis window shift is null.

Consequently, there is a periodicity on the relative position of an analysis window and signals according to translations of the analysis window. Since all shifts α_n are normalized with respect the symbol period T_s, this periodicity restricts the study to the interval $[0 \quad 1[$. Then, we put: $\langle x\rangle \equiv x \; modulo \; 1$, $x\in\mathbb{R}$. By setting: $\tau_n = \frac{T_{d_n}}{T_s}$, led us to write: $\alpha_n' = \langle d_f - \tau_n\rangle$, $n=0,\cdots,N_u^i-1$, with α_n' being the induced values. Hence, the FSNB-based criterion F can be rewritten as

$$F(d_f) = 1 + \sum_{n=0}^{N_u^i-1}\left\{\langle d_f-\tau_n\rangle^2 - \langle d_f-\tau_n\rangle\right\} \tag{10}$$

Since transmission delays τ_n remain constant during the synchronization process, (10) only depends on the analysis windows shifts d_f. Then, in the sequel, we analyse the criterion F and highlight that each signal is synchronized when the analysis window shift is equal to its corresponding transmission delay.

A. Criterion F synchronization peaks study

Let us recall that a peak is a point of a curve from which while moving by lower or higher values, the curve is always decreasing. And, as suggested in the discussion above, synchronization peaks should be at points $d_f = \tau_n$. This implies to examine criterion monotony in adjacent intervals: $]\tau_{n-1} \quad \tau_n]$ and $[\tau_n \quad \tau_{n+1}[$. However for $n=0$, or for $n=N_u^i-1$, intervals $]\tau_{-1} \quad \tau_0]$ and $[\tau_{N_u^i-1} \quad \tau_{N_u^i}[$ do not exist.

Thus, to take into account the periodicity of the configuration induced by analysis window shifts, let us put

$$\tilde{\tau}_n = \tau_{\langle n\rangle_{N_u^i}}, \quad \langle n\rangle_{N_u^i} \equiv n \quad modulo \quad N_u^i \tag{11}$$

Hence, (10) can be expressed as :

$$F(d_f) = 1 + \sum_{n=0}^{N_u^i-1}\left\{\langle d_f-\tilde{\tau}_n\rangle^2 - \langle d_f-\tilde{\tau}_n\rangle\right\} \tag{12}$$

Equation (12) appears to be a convex quadratic function of d_f over any interval $[\tilde{\tau}_n \quad \tilde{\tau}_{n+1}[$. The synchronization peaks of (12) are located at points $d_f = \tilde{\tau}_n$, $n=0,\cdots,N_u^i-1$, as illustrated on Fig. 2, where $N_u^i = 4$, $\tilde{\tau}_0 = 0$,$\tilde{\tau}_1 = 0.1509$,$\tilde{\tau}_2 = 0.3784$,$\tilde{\tau}_3 = 0.6979$.

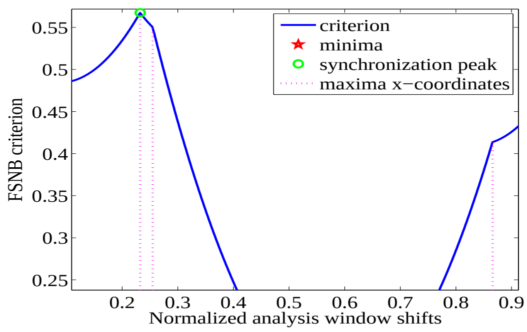

Figure 3. FSNB Criterion: case of the masking of synchronization peaks, $N_u^i = 4$

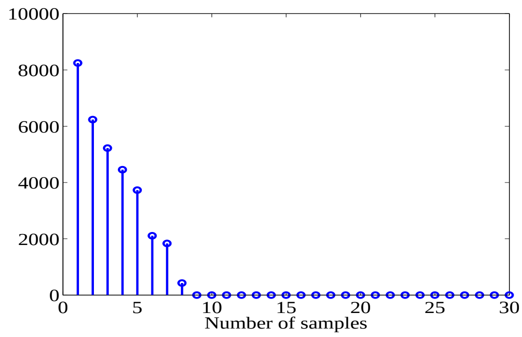

Figure 4. 30 first Matrix $\mathbf{R}$ eigenvalues before the sycnhronization, $N_u^i = 4$

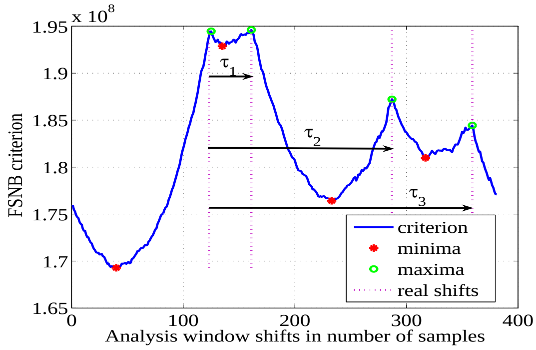

Figure 5. Experimental FSNB-based criterion F, $N_u^i = 4$

However, from (12), we demonstrated that according to transmissions delay choice, a local minimum of (12) may not belong to the interval $[\tilde{\tau}_n \quad \tilde{\tau}_{n+1}[$, in this case, it could not exist a synchronization peak for $d_f = \tilde{\tau}_n$: it is the phenomenon of synchronization peaks masking, as illustrated on Fig. 3. Hence, by studying the criterion behaviour in the vicinity of points such that $d_f = \tilde{\tau}_n$, leads to the finding of the following condition:

$$\tilde{\tau}_{n-1} < \frac{n}{N_u^i} < \tilde{\tau}_n < \frac{n+1}{N_u^i} < \tilde{\tau}_{n+1} \tag{13}$$

Equation (13) gives the necessary and sufficient condition of the existence of synchronization peaks. Moreover, one easily checks that in the particular case where the transmissions delays are equi-spaced, i.e. $\tilde{\tau}_n = \frac{n}{N_u^i}$, or that of downlink, i.e. $\tilde{\tau}_n = 0$, $n = 0, \cdots, N_u^i$, (13) is always satified. When the synchronization process has been performed, sequences can be identified as detailed in the following subsection.

B. Correlation matrix after synchronization and sequences identification

When a user is synchronized, e.g. $d_f = \tilde{\tau}_0$, which corresponds to $\alpha_0 = 0$, the correlation matrix becomes

$$\mathbf{R} = \sigma_b^2 \left\{ \beta \mathbf{v}_0 \mathbf{v}_0^* + \beta \sum_{n=1}^{N_u^i - 1} \{(1-\alpha_n)\mathbf{v}_n^0(\mathbf{v}_n^0)^* + \alpha_n \mathbf{v}_n^{-1}(\mathbf{v}_n^{-1})^*\} + \mathbf{I} \right\} \tag{14}$$

Equation (14) highlights that, once a user is synchronized, the correlation matrix has a maximum eigenvalue which associated vector contains the corresponding spreading sequence (apart from the effects of the global transmission filter), $2(N_u^i - 1)$ eigenvalues generated by the other users, and $M - 2N_u^i + 1$ eigenvalues equal, on average, to the noise power.

Then this process is performed in an iterative way so as to get the N_u^i largest eigenvalues, which the N_u^i associated vectors contain the corresponding spreading sequences. Moreover, the case of downlink is particular, since all users are transmitted simultaneously. Hence, when users are synchronized, the correlation matrix becomes:

$$\mathbf{R} = \sigma_b^2 \left\{ \beta \sum_{n=0}^{N_u^i - 1} \mathbf{v}_n \mathbf{v}_n^* + \mathbf{I} \right\} \tag{15}$$

The correlation matrix (15) has N_u^i largest eigenvalues which associated eigenvectors contain sequences (apart from the effects of the total transmission filter), and $M - N_u^i$ eigenvalues equal, on average, to the noise power. Then, thanks to linear algebra techniques, the sequences are estimated, and the symbols are demodulated, as detailed in [2], [3]. The subsection below deals with the determination of the number of users within each group of users transmitting at the same data rate.

C. Determination of the number of interfering users

This process is carried out simultaneously with synchronization. Indeed, when some peaks are masked, the knowledge of the number of users allows one to precisely re-estimate their positions, and to synchronize the corresponding users.

In uplink, N_u^i is equal to the number of synchronization peaks. However, as discussed in subsection IV-A, some peaks can be masked. Hence, one can use the cyclic Pastd algorithm proposed in [5]. In downlink, the number of interfering users N_u^i, is directly equal to the number of largest eigenvalues of the correlation matrix (15).

Finally, once the positions of all synchronization peaks are known, the differences between their positions and that corresponding to the user which normalized shift is closest to 0 or 1, is equal the corresponding transmissions delay.

V. Simulation Results

Simulations were carried out in uplink with $N_u^i = 4$ signals of $170\mu s$, each of them spread by a complex Gold sequence of length $L = 127$. The chip frequence was $F_c = 100$ Mhz, the initial sampling frequence was $F_e = 300$ Mhz, the SNR was $-5dB$ at the detector input, i.e., only the MAI noise is considered. The initial number of window was $N = 58$, with a duration of $550\mu s$, and the number of samples was 32768.

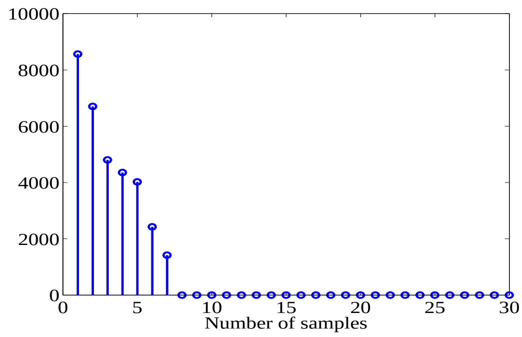

Figure 6. 30 first Matrix $\mathbf{R}$ eigenvalues after the sycnhronization, $N_u^i = 4$

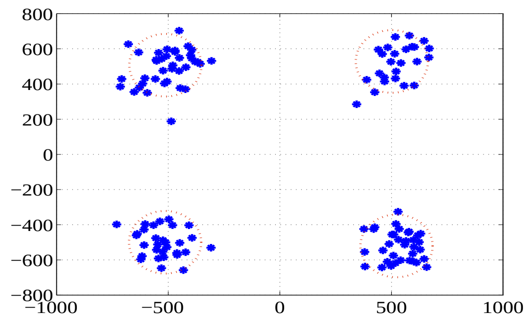

Figure 8. Estimated symbols

The symbols belong to a $QPSK$ constellation. The symbol period estimed as described in [6] was $T_s = 1.27\mu s$. So, it was set: $T_F = T_s$, and normalized arbitrary analysis windows shifts at the beginning of the process was set: $\alpha_0 = 0.3228$, $\alpha_1 = 0.4226$, $\alpha_2 = 0.7533$, $\alpha_3 = 0.9423$.

Fig. 4 illustrates the matrix $\mathbf{R}$ eigenvalues before the synchronization, and highlights $2N_u^i = 8$ eigenvalues as expected.

Fig. 5 depicts the experimental FSNB criterion. In agreement with the theory, Fig. 5 evidences 4 peaks which positions give desynchronization times in number of samples, and which number is equal to the number of active users. We obtain on Fig. 5: $\alpha_0 \approx 0.3281$, $\alpha_1 = 0.4226$, $\alpha_2 = 0.7533$ and $\alpha_3 = 0.9449$; $\tilde{\tau}_1 = 0.0999$, $\tilde{\tau}_2 = 0.4252$, and $\tilde{\tau}_3 = 0.6221$. These values are very close to the real ones.

Once a user is synchronized, the matrix $\mathbf{R}$ evidences a largest eigenvalue which associated eigenvector contains the sequence used to spread the synchronized user signal, and $2N_u^i = 6$ ones corresponding to the others signals as shown on Fig. 6.

Fig. 7 shows the bipolar estimated sequence that perfectly corresponds to the one used at the transmitter side. Then, by correlation, transmitted symbols are recovered as illustrated on Fig. 8; the constellation is similar to a $QPSK$ modulation, and impact dispersions are due to the low SNR.

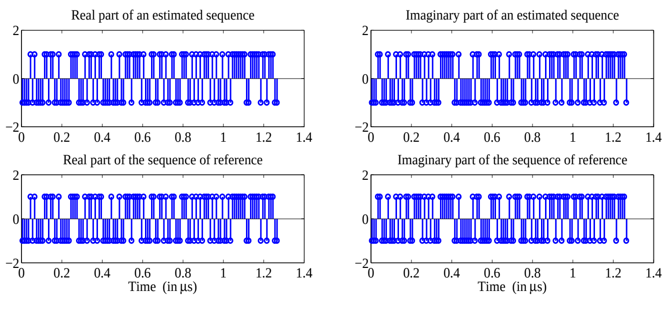

Figure 7. Bipolar estimated sequence

VI. Conclusion

In this paper, we extended to the multiuser case, the blind synchronization method based on the Frobenius Square Norm Behaviour (FSNB) of the correlation matrix previously proposed in [1]–[3], and explained the masking of some synchronization peaks that is due to the FSNB-based criterion definition itself.

Moreover, we proved that the number of synchronization peaks is equal to that of the interfering users within a group. Furthermore, we showed that in downlink, the number of interfering users is equal to that of the largest eigenvalues of the correlation matrix after the synchronization.

Acknowledgement

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