# Blind Multiuser Detection in Multirate CDMA Transmissions Using Fluctuations of Correlation Estimators

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*Abstract*— **This paper deals with the problem of blind multiuser detection in multirate direct-sequence code division multiple access (DS-CDMA). Direct-Sequence Spread Spectrum (DS-SS) signals are well-known for their low probability of interception: their statistics are similar to those of a noise; furthermore, they are usually transmitted below the noise level. Here, the method based on the fluctuations of autocorrelation estimators, previously described in a single user context [1], is extended to multiusers. From the variable spreading length technique, we will evidence an increase of these periodical fluctuations and, on average, a relationship between their peak amplitudes and sequence lengths. A theoretical analysis will show that detection is possible, even with a very low signal-to-noise ratio (SNR) at the detector input. This approach will be illustrated with experimental results. The assessment of symbol periods allows one to start the synchronization process proposed recently in [2], [3], which also permits the determination of the number of interfering users in each group of users transmitting at the same symbol period, thus, conversely at the same data rate.**

## I. Introduction

DS-CDMA systems are nowadays of increasing importance in wireless cellular communications because of their inclusion in most of the proposals about both terrestrial and satellite-based standards for third-generation (3G) wireless networks [4], [5]. One of the salient features of 3G cellular systems is the capability of supporting transmission data as diverse as voice, packet data, low-resolution video, compressed audio, etc... Since these heterogeneous services produce digital information streams with different data rates, their implementation requires the use of multirate CDMA systems where each user may transmit his data at one among a set of available data rates. An easy way to view the multirate CDMA transmission is to consider the variable spreading length (VSL) technique where all users employ sequences with the same chip period; moreover, the data rate is tied to the length of the spreading code of each user.

Many blind schemes and algorithms have been devised to either improve the performance of a CDMA receiver in a multirate multiuser context or reduce its complexity. Some prior knowledge of user parameters, e.g. the signature waveform [6], the processing gain, the code of a group of active users [7], the chip rate [8], [9] is always assumed, but its nature depends on the technique employed. Here, we propose a blind detection method requiring no prior knowledge about the transmitter. It is typically the case in blind signals interception in the military field or in spectrum surveillance.

These considerations led us to extend the detection scheme initially proposed in [1] to multirate CDMA transmissions. This method should also allow a blind multi-standard detection through a differentiation of various standards based on an analysis of their set of available data rates [10].

This report is organized as follows: Section II will introduce the signal model and assumptions made. Section III will describe our blind-detection approach. Section IV will deal with the theoretical analysis of the proposed method. The simulations results will be detailed in Section V, and our conclusions will be drawn in Section VI.

## II. Signal modeling and assumptions

Let us consider the general case of asynchronous DS-CDMA system where each user can transmit at one out of $S$ available data rates $R_0 < R_1 < \cdots < R_{S-1}$. By denoting $N_u^i$ the number of active users transmitting at $R_i$ and $N_u$ the total number of users such that $\sum_{i=0}^{S-1} N_u^i = N_u$, the complex equivalent of the received signal can be expressed as:

$$y(t) = \sum_{i=0}^{S-1} \sum_{n=0}^{N_u^i-1} \sum_{k=-\infty}^{+\infty} a_{n,i}(k) h_{n,i}(t - kT_{s_i} - T_{d_{n,i}}) + b(t) \quad (1)$$

where $h_{n,i}(t) = \sum_{k=0}^{L_i-1} c_{n,i}(k) p_i(t - kT_c)$. In (1), the subscript $(\cdot)_{n,i}$ refers to the $n^{th}$ user transmitting at $R_i$, denoted throughout this report as the $(n,i)^{th}$ user. Accordingly:

- $a_{n,i}(k)$ are the baseband symbols of variance $\sigma^2_{a_{n,i}}$ for the $(n,i)^{th}$ user, whereas $p_i(t)$ is the convolution of the transmission filter, channel filter (which takes into account channel echoes, fading, multipaths and jammers) and receiver filter for each rate.
- The term $h_{n,i}(t)$ is a virtual filter corresponding to the convolution of all filters of the transmission chain with the spreading sequence $\{c_{n,i}(k)\}_{k=0\cdots L_i-1}$, where $L_i$ is the spreading factor for the $(n,i)^{th}$ user.

- Because of the VSL technique, the symbol period $T_{s_i}$ for the users transmitting at the rate $R_i$ is tied to the common chip period $T_c$: $T_{s_i} = L_i T_c$, and $s_{n,i}$ stands for the $(n,i)^{th}$ signal.
- The term $T_{d_{n,i}}$ is the corresponding transmission delay for the $(n,i)^{th}$ user; it is assumed to satisfy: $0 \leqslant T_{d_{n,i}} < T_{s_i}$ and to remain constant during the observation.
- $b(t)$ is a centered white Gaussian noise of variance $\sigma_b^2$.
- Signals are assumed to be independent, centered, noise-uncorrelated and received with the same power: $\sigma^2_{s_{n,i}} = \sigma^2_{s_{0,0}}$, for all $(n,i)$.
- Finally, the $SNR$ (in $dB$) at the detector intput is negative (signal hidden in the noise).

By denoting $s(t)$ the global noise-unaffected received signal, (1) can be rewritten as:

$$y(t) = s(t) + b(t) = \sum_{i=0}^{S-1} \sum_{n=0}^{N_u^i - 1} s_{n,i}(t - T_{d_{n,i}}) + b(t) \quad (2)$$

## III. Proposed approach

The case under study is a non-cooperative context with no prior information available (unknown spreading sequence, symbol period,...). The basic principle of any detection method, whatever the application, is to take profit of the difference between the statistical properties of a signal and those of noise. Indeed, in some simple applications, the great difference between signal and noise frequencies permits the detection of a signal by filters. But, here, the application is far more complex: indeed, to reduce the probability of interception, a spread spectrum signal has to be very similar to noise, e.g. its autocorrelation and that of a white noise are both close to a Dirac function because of the pseudo-random sequence. The novelty of the proposed approach is to be based not on the autocorrelation, but on the fluctuations of its estimators. To compute the fluctuations, let us divide the received signal into temporal windows and denote $T_F$ the window duration and $M$ the number of windows. Then, an autocorrelation estimator is applied to each window prior to the computation. By using the $m^{th}$ window, let us compute an estimation of the autocorrelation for any signal $r(t)$:

$$\widehat{R}^m_{rr}(\tau) = \frac{1}{T_F} \int_0^{T_F} r_m(t) r_m^*(t - \tau) dt \quad (3)$$

where $r_m(t)$ is the signal sample over the $m^{th}$ window. The use of $M$ windows allows us to estimate the second-order moment of the estimated correlation $\widehat{R}^m_{rr}(\tau)$ as:

$$\Phi(\tau) = \widehat{E}\left\{ |\widehat{R}_{rr}(\tau)|^2 \right\} = \frac{1}{M} \sum_{m=0}^{M-1} |\widehat{R}^m_{rr}(\tau)|^2 \quad (4)$$

where $\widehat{E}(\cdot)$ is the estimated expectation of $(\cdot)$. Hence, $\Phi(\tau)$ is a measure of the fluctuations of $\widehat{R}^m_{rr}(\tau)$. Applying (3) to (2) with the assumptions of independent, centered and noise-uncorrelated signals leads to the following equations for both uplink and downlink transmissions:

$$\widehat{R}_{ss}(\tau) = \sum_{i=0}^{S-1} \sum_{n=0}^{N_u^i - 1} \widehat{R}_{s_{n,i}s_{n,i}}(\tau) \quad (5)$$

$$\widehat{R}_{yy}(\tau) = \widehat{R}_{ss}(\tau) + \widehat{R}_{bb}(\tau). \quad (6)$$

where $\widehat{R}_{s_{n,i}s_{n,i}}(\tau)$ and $\widehat{R}_{bb}(\tau)$ are the estimates of the $(n,i)^{th}$ noise-unaffected signal and that of the noise autocorrelation fluctuations, respectively. Indeed, since the fluctuations are computed from many randomly-selected windows, they do not depend on the signals relative delays $T_{d_{n,i}}$.

## IV. Theoretical analysis

To evidence the efficiency of the proposed method, even at very low $SNR$, let us successively investigate the contributions of noise and signal through the analysis of the second-order moment of the autocorrelation estimator.

### A. Contribution of the noise

At first, let us consider the additive noise alone, i.e. with no hidden spread spectrum signal. Indeed, since the analysis is focused on the fluctuations of autocorrelation estimators, but not on the autocorrelation itself, only the fluctuations due to the additive noise are uniformly distributed over all values of $\tau$. Depending on the spreading sequence properties, the Multiple Access Interference (MAI) noise generates rather null, or very low, incoherent fluctuations. Since the noise is random, the fluctuations of the autocorrelation estimator, denoted $\Phi_b(\tau)$, are also random. Let us characterize them by their mean $(m_{\Phi_b})$ and standard deviation $(\sigma_{\Phi_b})$. If the frequency response of the receiver filter is flat in $[-W/2, +W/2]$ and zero outside, like in [1], we get:

$$m_{\Phi_b} = \frac{\sigma_b^4}{W T_F} \quad (7)$$

where $\sigma_b^2$ is the additive noise variance. The standard deviation of $\Phi_b(\tau)$ is given by:

$$\sigma_{\Phi_b} = \sqrt{\frac{2}{M}} \frac{\sigma_b^4}{W T_F} \quad (8)$$

### B. Contribution of the signal

Let us, now, consider only the global spread spectrum signal to show that, on average, high amplitudes of the fluctuations of the autocorrelation estimator, denoted $\Phi_s(\tau)$, are obtained for each $\tau$ multiple of each symbol period $T_{s_i}$, $i = 0, \cdots, S-1$. Since the symbol periods are different, let us denote $\Phi_i(\tau)$ the fluctuations of the autocorrelation estimator of the $N_u^i$ signals $s_{n,i}$ transmitting at the same data rate $T_{s_i}$. Let us also term $m_{\Phi_i}$ the mean value of the fluctuations $\Phi_i(\tau)$ for each value $\tau$ multiple of $T_{s_i}$; it leads to:

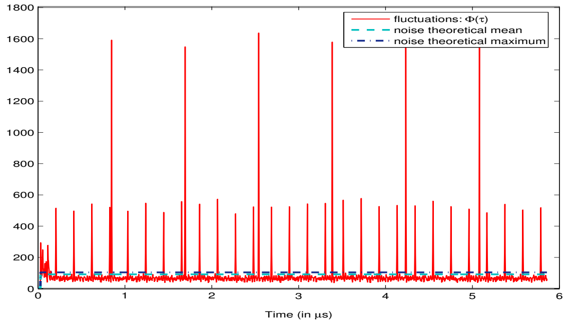


Figure 1. Fluctuations $\Phi(\tau)$, $N_u^0 = 2$, $N_u^1 = 2$, $SNR = -5 \ \ dB$.

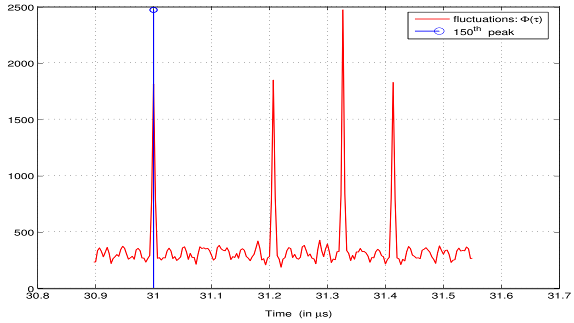


Figure 2. Estimation of $T_{s_0}$.

$$\Phi_i(\tau) = m_{\Phi_i} \cdot pgn_{T_{s_i}}(\tau) \qquad (9)$$

where $pgn_{T_{s_i}}(\tau) = \sum_{k=-\infty}^{+\infty} \delta(\tau - kT_{s_i})$, and the function $\delta(\tau)$ is defined as:

$$\delta(\tau) = \begin{cases} 1, \ if \ \tau = 0 \\ 0, \ if \ \tau \neq 0 \end{cases} \qquad (10)$$

Consequently, the signals being independent and centered by assumption, the fluctuations $\Phi_s(\tau)$ of the global noise-unaffected signal, for each value $\tau$ multiple of $T_{s_i}$, can be expressed as:

$$\Phi_s(\tau) = \sum_{i=0}^{S-1} \Phi_i(\tau) = \sum_{i=0}^{S-1} m_{\Phi_i} \cdot pgn_{T_{s_i}}(\tau) \qquad (11)$$

Let us, now, extend the results about the single user context [1] to the multirate CDMA one; since the signals are assumed to be received with the same power (equal to $\sigma^2_{s_{0,0}}$, for example) and $T_{s_i} = L_i T_c$, for each value $\tau$ multiple of $T_{s_i}$, the average value amplitude $m_{\Phi_i}$ of the estimator fluctuations in each group of $N_u^i$ users transmitting at the same data rate can be written as:

$$m_{\Phi_i} = N_u^i \frac{T_{s_i}}{T_F} \sigma^4_{s_{0,0}} = N_u^i \frac{L_i T_c}{T_F} \sigma^4_{s_{0,0}} \qquad (12)$$

### *C. Remarks and analysis*

At this point, it is worth noting that, on average, (12) shows an increase of fluctuation amplitudes concomitant with that of the number of users. Moreover, it indicates that, on average, the amplitude of fluctuations is proportional to the length of the sequences used.

Therefore, the longer the sequence is, the higher the peaks of autocorrelation fluctuations are, and thus the biggest amplitude is usually exhibited by the fluctuations of the $N_u^0$ users transmitting at the lowest data rate. This approach is thus, a powerful tool to estimate symbol periods.

it allows one to differentiate the various transmitted data rates and, thus, to distinguish between the different standards. Indeed, (12) shows that, for each multiple of each $T_{s_i}$, high fluctuations of the autocorrelation estimator are obtained, and their amplitudes are tied to $L_i$ and $N_u^i$. So, in presence of a spread spectrum signal, the fluctuations curve highlights equispaced peaks whose average spacing corresponds to the estimated period symbol $\tilde{T}_{s_i}$. It ensues that, in each group of $N_u^i$ users, one can start the blind synchronization process proposed in [2], [3] and determine the number of interfering users $N_u^i$ in each group, as illustrated in section V.

Let us assume that: $T_{s_0} = \omega_i T_{s_i}$, $\forall i = 1, \cdots, S-1$, where $\{\omega_1, \cdots, \omega_{S-1}\}$ is a set of increasing integers, i.e. all data rate are multiple of the lowest one. Therefore, whenever $\tau = k \cdot \omega_i T_{s_0}$, $k \in \mathbb{N}^*$, a superimposition of the fluctuations peaks ensues from (11), but according to (12) their amplitudes are diffemt and tied to each symbol period $T_{s_i}$. Hence, the estimation of symbol periods is unaffected.

In addition, let us denote $\rho$, the $SNR$ of a given signal such that $\sigma^2_{s_{0,0}} = \rho\sigma^2_b$, and $\mathcal{P}_t = N_u \sigma^2_{s_{0,0}} + \sigma^2_b$, the total received power. Their use in (12) leads to the normalized mean amplitudes of fluctuations:

$$m_{\Phi_i} = N_u^i \frac{T_{s_i}}{T_F} \left( \frac{\mathcal{P}_t}{N_u + \rho^{-1}} \right)^2 \qquad (13)$$

Thanks to this normalization in (13), in practice the calculation capacities are not overflowed. Moreover, on average the fluctuations amplitude is decreased, for each value $\tau$ multiple of $T_{s_i}$, for fixed $N_u^i$ and $N_u$ when the $SNR$ at the detector input is very low. Since $N_u^i < N_u$, for fixed $\rho$, (13) highlights a decrease of the fluctuations amplitude when the total number $N_u$ of users is increasing.

## V. Simulation Results

To determine whether a spread spectrum signal is hidden in the noise, let us compare $\Phi(\tau)$ (fluctuations of the global noise-affected signal) with theoretical mean $m^{(th)}_{\Phi_b}$ and maximum of the noise fluctuations $\Phi_b(\tau)$. Together (7) and (8) show that $m_{\Phi_b}$ and $\sigma_{\Phi_b}$ are affected by only the noise variance

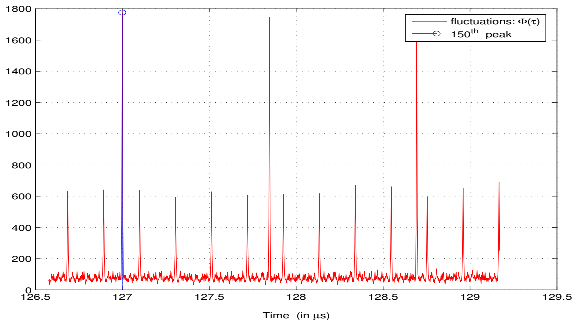


Figure 3. Estimation of $T_{s_1}$.

and the simulation parameters, and thus:

$$\begin{cases} m_{\Phi_b}^{(th)} = m_{\Phi_b} \\ \sigma_{\Phi_b}^{(th)} = \sigma_{\Phi_b} \end{cases} \quad (14)$$

Hence, the theoretical maximum of the noise fluctuations, i.e. the detection threshold, is deduced from (14):

$$m_{\Phi_b}^{(th)} + 3 \cdot \sigma_{\Phi_b}^{(th)} \quad (15)$$

The aim of theoretical calculation is to predict the average and the maximum of the fluctuations curve when there is no signal hidden in the noise. Whenever a spread spectrum signal is hidden in the noise, the average of the curve deviates from the theoretical average, and in particular, the maximum of the curve is above the theoretical maximum of noise fluctuations. It is precisely this difference between the theoretical predictions and the values of the obtained fluctuations that permits the detection of the signal hidden in the noise. When there is only noise, this difference is very weak.

### A. Symbol periods estimation and standards differenciation

Simulations were, thus, carried out on assuming a processing gain of $L_0 = 31$ and $L_1 = 127$ (Complex GOLD sequences), and on fixing the common chip period, $F_c = 150$ $MHz$, the sampling period, $F_e = 300$ $MHz$, the window duration $T_F = 2$ $\mu s$, and the number of windows, $M = 300$. For the sake of simplicity, we set $N_u^0 = 2$, $N_u^1 = 2$, hence $N_u = 4$, and $SNR \approx -5$ $dB$ (we assume only the MAI noise). The general case of uplink is considered in this report.

Fig.1 illustrates the fluctuations of the autocorrelation estimator $\Phi(\tau)$ versus $\tau$ (in $\mu s$). The curve clearly highlights two sets of equispaced peaks of different amplitudes. It means that two sets of spread spectrum signals transmitting at two different rates are hidden in the noise; the theoretical analysis, described above, demonstrated us that these peaks resulted from multiple symbol periods, which allowed the determination of these symbol periods (in $\mu s$): $\widetilde{T}_{s_0} = 0.2061$, $\widetilde{T}_{s_1} = 0.8467$. In this configuration, simulations showed that detection was possible up to a $SNR = -25$ $dB$ at the detector input.

TABLE I
DETECTOR PERFORMANCES.

| Sequences (Length) | Average MSE | Standard deviation |
|---|---|---|
| OVSF (32) | $3.5685\ 10^{-5}$ | 0.0059 |
| WALSH (16) | $3.0162\ 10^{-6}$ | 0.0017 |
| OVSF+WALSH (32, 16) | $1.2\ 10^{-2}$ | 0.1095 |
| OVSF+WALSH+GOLD (32, 16, 127) | $8.1\ 10^{-3}$ | 0.09 |

It is worth underlining that detection remains possible for far much lower $SNRs$ on condition to use more numerous analysis windows, but the computation time is far much longer.

To re-estimate these values, for each set of autocorrelation fluctuations, let us look for a maximum near the farthest peak on the right side. For example, let us consider the $150^{th}$ peak and check that: $150 \times \widetilde{T}_{s_0} \approx 31\mu s$ and $150 \times \widetilde{T}_{s_1} \approx 127\mu s$. Hence, there are maxima at the $150^{th}$ peak in agreement with Figs. 2 and 3.

Table I gives the detector average relative MSE (Mean Square Error of estimated periods) for $SNRs$ (in $dB$) in the range $[-14 \quad -5]$ for different sequences (4 users for each type of sequences) as well as the standard deviation of the MSE. The MSE is usually negligible even at very low $SNRs$, and the dispersion of estimated symbol periods around the real ones is very low. It proves the efficiency and power of our method for the blind detection of multiusers. Moreover, depending on sequence properties, this error tends to zero, especially at high $SNRs$, which is the case for civilian communications.

### B. Determination of the number $N_u^i$ of interfering users in each group

As described in [2], [3], the knowledge of the estimated symbol period, $\widetilde{T}_{s_i}$, allows one to perform the synchronization process in each group of users transmitting at the same data rate; one should note that the window duration $T_F$ is reajusted over the synchronization process so that: $T_F = \widetilde{T}_{s_i}$.

The fast and efficient blind synchronization criterion developed in [2], [3] is based on the behavior of the FROBENIUS square norm of submatrices that scan the intercepted signal correlation matrix.

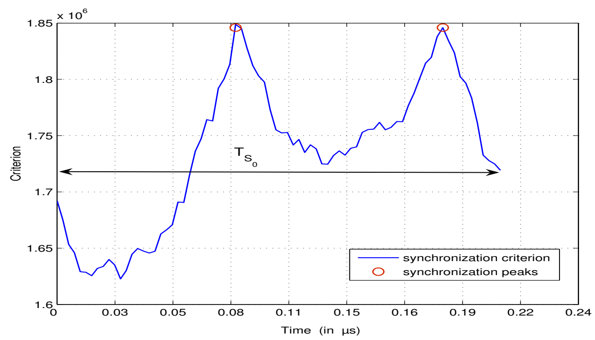


Figure 4. Synchronization criterion in uplink, $N_u^0 = 2$, $SNR = -5$ $dB$.

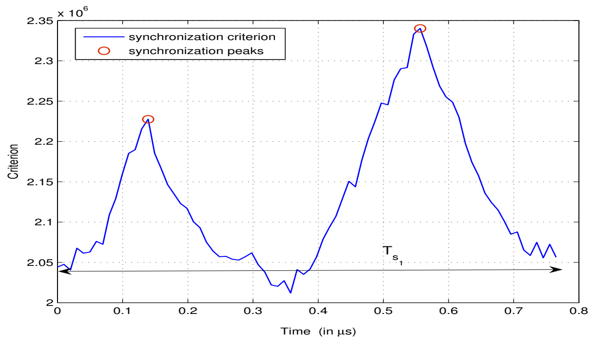


Figure 5. Synchronization criterion in uplink, $N_u^1 = 2$, $SNR = -5\ dB$.

In downlink, since the signals are transmitted simultanously, the criterion curve presents only one synchronization peak. Hence, the number of largest eigenvalues of one of the submatrices extracted from the global signal correlation matrix is equal to the number $\widetilde{N_u^i}$ of interfering users in the corresponding group.

In uplink, the criterion curve obtained in the interval $[0,\ \widetilde{T_{s_i}}]$ presents several peaks, and their number is equal to the number $N_u^i$ of interfering users transmitting at the same data rate:

- In the first group of users, i.e. $T_{s_0} = \widetilde{T_{s_0}} = 0.2061\ \mu s$, Fig. 4 highlights two synchronization peaks. Hence, the number of users is equal to $\widetilde{N_u^0} = N_u^0 = 2$, as expected.
- In the second group, i.e. $T_{s_1} = \widetilde{T_{s_1}} = 0.8467\ \mu s$, Fig. 5 also highlights two synchronization peaks. Consequently, there are two users transmitting in this group. So, the number of users is equal to $\widetilde{N_u^1} = N_u^1 = 2$.

Thus, simulation results show that our method allows one to detect the different data rates, and, especially, to determine them via the symbol periods estimation. Then, combined with the blind synchronization method that we recently proposed in [2], [3], our approach allows one to determine the number of interfering users transmitting at the same data rate, even with a very low SNR at the detector input.

## VI. Conclusions

The scheme developed, here, for blind-detection of multiusers in multirate CDMA transmissions proved its efficiency and power both in uplink and downlink. We also highlighted its ability to estimate symbol periods. Indeed, despite the high similarity between the autocorrelation of a DS-SS signal and that of a noise, we demonstrated that the fluctuations of correlation estimators are higher when spread spectrum signals are hidden in the noise.

The average spacing evidenced on the fluctuations curves between the high, equispaced peaks allowed the determination of symbol periods. We also evidenced a relationship between fluctuation amplitudes, sequence lengths and the number of users transmitting at the same data rate. It allowed one to determine the number of user groups transmitting at the same data rate and thus to distinguish one standard from others as previously suggested in [10]. As proposed in [2], [3], the number of users within a group can be found through the synchronization process.

Moreover, this blind detection scheme is applicable to passband transmissions with unknown carrier frequencies. In the single user context, it was already implemented in an operational system by scanning all the available frequency bands and recovering the carrier frequencies as done in classical transmissions.

## VII. Acknowledgement

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