MINIMUM BER DIAGONAL PRECODER FOR MIMO SYSTEMS

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Abstract:
We report on a simplified representation of Multi-Input Multi-Output (MIMO) systems and propose a Minimum Bit Error Rate (MBER) diagonal precoder. Assuming that channel information can be available at the transmitter, we show that the proposed representation decouples the MIMO channel into parallel subchannels, which greatly facilitates and speeds up further processing. Using traditional criteria such as the minimum mean square error (MMSE) and the maximum capacity, eigen diagonal precoders and decoders can be obtained, leading to the same transmit and receive filters as those reported in the literature but in a simpler and faster way. A new diagonal precoder is also derived using the minimum bit error rate (MBER) criterion and compared to the others in term of bit error rate and achieved capacity. An approximation of the MBER precoder (AMBER) is finally proposed, whose performances remain close to the optimal in spite of its low complexity.

I INTRODUCTION

Multi-Input Multi-Output (MIMO) digital transmission systems currently retain more and more attention due to the very high spectral efficiencies they can achieve. Most existing systems such as spatial multiplexing [3] or space-time coding [11] assume no channel knowledge at the transmitter. However, in many wireless applications, feedback does exist (e.g., symmetric or asymmetric duplex transmissions), and channel information can be made available at the transmitter. Indeed, only a very small data rate is used to provide channel information to the transmitter. The question, then, is how to take profit of this information to globally optimize the transmission system.

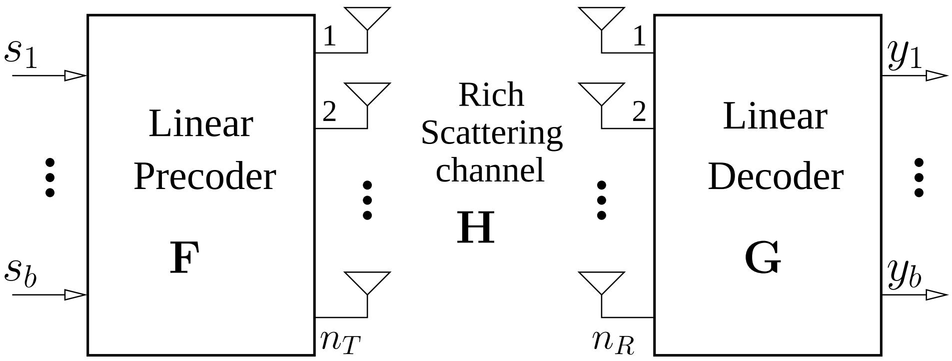


Figure 1: MIMO System with linear precoder and decoder

Performances can be improved either by antenna selection [4] or precoders design in order to allocate power among the transmit antennas. In [8] and [9] Sampath *et al* have globally optimized the transmission scheme by designing optimum precoder and decoder using the minimum mean square error (MMSE) criterion. An interesting property of their work is the diagonality of their equivalent channel after optimization. This diagonality can also be obtained by the singular value decomposition (SVD) method, especially used to maximize the channel capacity, which leads to the well-known water-filling (WF) solution [2, 5]. In this paper, a similar technique is employed to provide a simplified diagonalized MIMO system which can be easily optimized by any criterion (not only the channel capacity maximization). In addition, this diagonality results in an interesting complexity reduction, which enables one to use a maximum-likelihood (ML) receiver.

As the bit error rate (BER) can then be expressed in a quite simple way, we propose a new precoder based on the minimum BER (MBER) criterion. By comparisons with other criteria such as MMSE or the WF solution, we show that the use of the MBER criterion increases the system performances in term of error rate and achieved capacity for a given constellation. We also present an approximation of the MBER precoder (AMBER), whose results remain very close and which is faster because it does not need any optimization.

II SIMPLIFIED MIMO SYSTEM MODEL

Let us consider a MIMO system with n_R receive and n_T transmit antennas over which we want to achieve b independent data streams. Including a precoder matrix $\mathbf{F}$ and a decoder matrix $\mathbf{G}$, the basic system model is (Fig. 1):

$$\mathbf{y} = \mathbf{GHFs} + \mathbf{G}\nu \tag{1}$$

where $\mathbf{H}$ is the $(n_R \times n_T)$ channel matrix, $\mathbf{F}$ is the $(n_T \times b)$ precoder matrix, $\mathbf{G}$ is the $b \times n_R$ decoder matrix , $\mathbf{s}$ is the $(b \times 1)$ transmitted vector and ν is the $(n_R \times 1)$ noise vector. We assume : $E\{\mathbf{ss}^*\} = \mathbf{I}_b$, $E\{\nu\nu^*\} = \mathbf{R}$ and $E\{\mathbf{s}\nu^*\} = 0$.

Furthermore, if the available transmission power is noted p_0, the constraint below must be fulfilled: $trace\{\mathbf{FF}^*\} = p_0$.

Our first objective is to obtain a diagonal channel and a whitened noise in order to facilitate both the system analysis and the determination of the optimal precoder. We decompose the matrices $\mathbf{F} = \mathbf{F}_v\mathbf{F}_d$ and $\mathbf{G} = \mathbf{G}_d\mathbf{G}_v$, where the virtual precoder and decoder matrices $\mathbf{F}_v$ and $\mathbf{G}_v$ are designed to reach this first objective, such that the determination of precoder $\mathbf{F}_d$ and decoder $\mathbf{G}_d$ with respect to any criterion is greatly facilitated. They are obtained after successive matrix manipulations and transformations, such as the singular value decomposition (SVD), and are decomposed as follows: $\mathbf{F}_v = \mathbf{F}_1\mathbf{F}_2\mathbf{F}_3$ and $\mathbf{G}_v = \mathbf{G}_3\mathbf{G}_2\mathbf{G}_1$. The three different steps necessary to get $\mathbf{F}_v$ and $\mathbf{G}_v$ are resumed in Table 1.

step	i	method	$\mathbf{F}_i$	$\mathbf{G}_i$
noise whitening	1	EVD: $\mathbf{R} = \mathbf{Q}\Lambda\mathbf{Q}^*$	$\mathbf{F}_1 = \mathbf{I}_{n_T}$	$\mathbf{G}_1 = \Lambda^{-\frac{1}{2}}\mathbf{Q}^*$
channel diagonalization	2	SVD: $\mathbf{G}_1\mathbf{HF}_1 = \mathbf{A}\Sigma\mathbf{B}^*$	$\mathbf{F}_2 = \mathbf{B}$	$\mathbf{G}_2 = \mathbf{A}^*$
dimensionality reduction	3	$\mathbf{F}_v = \mathbf{F}_1\mathbf{F}_2\mathbf{F}_3$ $\mathbf{G}_v = \mathbf{G}_3\mathbf{G}_2\mathbf{G}_1$	$\mathbf{F}_3 = \begin{pmatrix} \mathbf{I}_b \\ 0 \end{pmatrix}$	$\mathbf{G}_3 = \begin{pmatrix} \mathbf{I}_b & 0 \end{pmatrix}$

Table 1: Steps to obtain the diagonal MIMO system in case of CSI at the transmitter.

In this new representation the model becomes:

$$\mathbf{y} = \mathbf{G}_d \mathbf{H}_v \mathbf{F}_d \mathbf{s} + \mathbf{G}_d \nu_v \quad (2)$$

where $\mathbf{H}_v = \mathbf{G}_v \mathbf{H} \mathbf{F}_v$ is the virtual diagonal channel and $\nu_v = \mathbf{G}_v \nu$ is the virtual noise, whose correlation matrix $\mathbf{R}_v = \mathbf{G}_v \mathbf{R} \mathbf{G}_v^* = \mathbf{I}_b$. $\mathbf{H}_v$ entries are linked to the singular values of the channel $\mathbf{H}$ and can be expressed as $\mathbf{H}_v = diag(\sigma_1, ..., \sigma_b)$ with positive elements ranked in the decreasing order.

III PRECODER DESIGNS

The matrices F_d and G_d are still to be determined, according to a given criterion. Since the channel is diagonal and the noise white, we restrict our search to diagonal matrices $F_d = diag\{f_i\}_{i=1}^b$ and $G_d = diag\{g_i\}_{i=1}^b$. Hence, the system can be seen as a set of b independent and parallel subchannels. ML detection can be then performed efficiently even for large values of b and important constellations.

The values of these matrices depend on the criterion to be optimized as illustrated hereafter. Our approach provides a fast way to find many results already available in the literature as well as new ones for the MBER precoder and its approximation.

III.1 Classical criteria

The most popular criterion for MIMO systems optimization aims at maximizing the information rate of the transmission and is known as the water-filling (WF) solution.

Reminding the basic hypothesis, the equivalent model (2) leads to the following capacity [12]:

$$C = \sum_{i=1}^{b} \log_2(1 + f_i^2 \sigma_i^2) \quad (3)$$

Using Lagrange multiplier μ, the criterion to be optimized under the constraint on total transmit power is then:

$$\mathcal{C}_{WF} = \sum_{i=1}^{b} \log_2(1 + f_i^2 \sigma_i^2) + \mu \left(\left(\sum_{i=1}^{b} f_i^2 \right) - p_0 \right) \quad (4)$$

Cancellation of $\frac{\partial \mathcal{C}_{WF}}{\partial f_i}$ provides the precoder elements expression:

$$f_i^2 = \left(\frac{p_0 + \sum_{k=1}^{b_\Psi} 1/\sigma_k^2}{b_\psi} - \frac{1}{\sigma_i^2} \right)^+ \quad (5)$$

where $(s)^+ = s \;$ if $s > 0$ and $(s)^+ = 0 \;$ if $s \leq 0$, and b_Ψ represents the integer until which this positivity condition remains true.

The minimization of the mean square error (MMSE) is another optimization method, especialy appreciated for its robustness and low complexity. We introduce here the weigthed MMSE criterion, that allows to get information on each eigen mode and to act on it, leading to various applications such as the quality of services or the equal-error transmission.

Thanks to the diagonal representation, the weighted mean square error is expressed as:

$$\sum_{i=1}^{b} E[w_i \left| (g_i \sigma_i f_i - 1) s_i + g_i \nu_{v_i} \right|^2] \quad (6)$$

where w_i for $i = 1, \ldots, b$ are positive weighting coefficient.

That leads to the following criterion to be minimized:

$$\mathcal{C}_{MSE} = \sum_{i=1}^{b} w_i (g_i^2 \sigma_i^2 f_i^2 - 2 g_i \sigma_i f_i + g_i^2 + 1) + \mu [(\sum_{i=1}^{b} f_i^2) - p_0] \quad (7)$$

Cancellations of $\frac{\partial \mathcal{C}_{MSE}}{\partial f_i}$ and $\frac{\partial \mathcal{C}_{MSE}}{\partial g_i}$ provide the precoder and decoder expressions:

$$f_i^2 = \left(\frac{w_i(p_0 + \sum_{k=1}^{b_\Psi} 1/\sigma_k^2)}{\sigma_i \sum_{k=1}^{b_\Psi} w_k/\sigma_k^2} - \frac{1}{\sigma_i^2} \right)^+ \text{ and } g_i = \frac{\sigma_i f_i}{(\sigma_i f_i)^2 + 1} \tag{8}$$

The presence of w_i in this expression allows to act on each mode. The expression of the precoder based on the classical (unweighted) MMSE criterion can be easily recovered taking the weighting coefficients $w_i = 1$ for $i = 1, \dots, b$.

Some constant data rate communication systems need a very secure transmission and same SNR ρ on each way, *i.e.* $\rho = \rho_i = \sigma_i^2 f_i^2$ for $i = 1, \dots, b$. This particular configuration leads to the following precoder expression:

$$f_i^2 = \frac{p_0}{\sigma_i^2 \sum_{k=1}^{b} 1/\sigma_k^2} \tag{9}$$

This equal-error precoder can be obtained from another point of view, considering the criterion which aims at maximizing the minimum eigenvalue of the post-processing $SNR(\mathbf{F}, \mathbf{G})$ matrix (see [10]). This criterion provides a lower bound for the minimum distance of the received symbol vector [10].

Same results as (5), (8) and (9) can be found in the literature [8, 9, 10] but are obtained here by a simple way since a diagonalization of the MIMO system is performed in a previous step.

III.2 MBER criterion

As the performance of practical systems depends strongly on the bit error rate and achieved capacities, we propose a new precoder based directly on the minimum BER, whose efficiency has been proved in [7] for another configuration. As an ML receiver is used, $\mathbf{G}_d$ has no impact on the minimization of the BER, so we choose for simplicity $\mathbf{G}_d = \mathbf{I}_b$.

The signal to noise ratio for each sub-channel is:

$$\rho_i = \sigma_i^2 f_i^2 \tag{10}$$

For a square M-QAM constellation ($M = 2^{2n}$), the BER is $P_e = \frac{1}{b} \sum_{i=1}^{b} P_{e,i}$ where the BER in sub-channel i is [6]:

$$P_{e,i} = \alpha_M \times erfc\sqrt{\beta_M \times \rho_i} \qquad \text{with } \alpha_M = \frac{2}{\log_2 M}(1 - \frac{1}{\sqrt{M}}) \text{ and } \beta_M = \frac{3}{2(M-1)} \tag{11}$$

Using Lagrange multiplier μ, we can express the precoder elements f_i as follows:

$$f_i^2 = \frac{1}{2\beta_M \sigma_i^2} W_0 \left(\frac{2\sigma_i^4 \alpha_M^2 \beta_M^2}{\mu^2 \pi b^2} \right) \tag{12}$$

where W_0 stands for Lambert's W function of index 0 [1]. The parameter μ can be iteratively computed by using the constraint of the total transmit power.

A simplified solution of the MBER design can be performed by approximating the Lambert's W function of index 0 by:

$$W_0(x) \simeq \log(x) - \log(\log(x)) \quad \text{for } x \gg 1 \tag{13}$$

By using this approximation in (12) and the constraint power we directly obtain the approximated MBER (AMBER) solution:

$$f_i^2 = \frac{a_i(1 - \sum_k A_k) + A_i \sum_k a_k}{\sum_k a_k} \text{ for } i = 1, \dots, b \tag{14}$$

where:

$$a_i = \frac{1}{2\beta_M \sigma_i^2} \tag{15}$$

$$A_i = a_i(\log(b_i) - \log(\log(b_i))) \text{ with } b_i = \frac{2\sigma_i^4 \alpha_M^2 \beta_M^2}{\pi b^2} \tag{16}$$

One should note here that, in the low SNR case, the approximation (13) does not hold any more; $\log(b_i)$ may be smaller than 1 and (16) can not be computed. The AMBER method then neglects the weakest subchannel, just as MMSE or WF solutions, and (14) is computed over the remaining modes.

This AMBER method is the fastest one among those presented in this paper, because it needs neither optimization nor μ search.

IV SIMULATION RESULTS

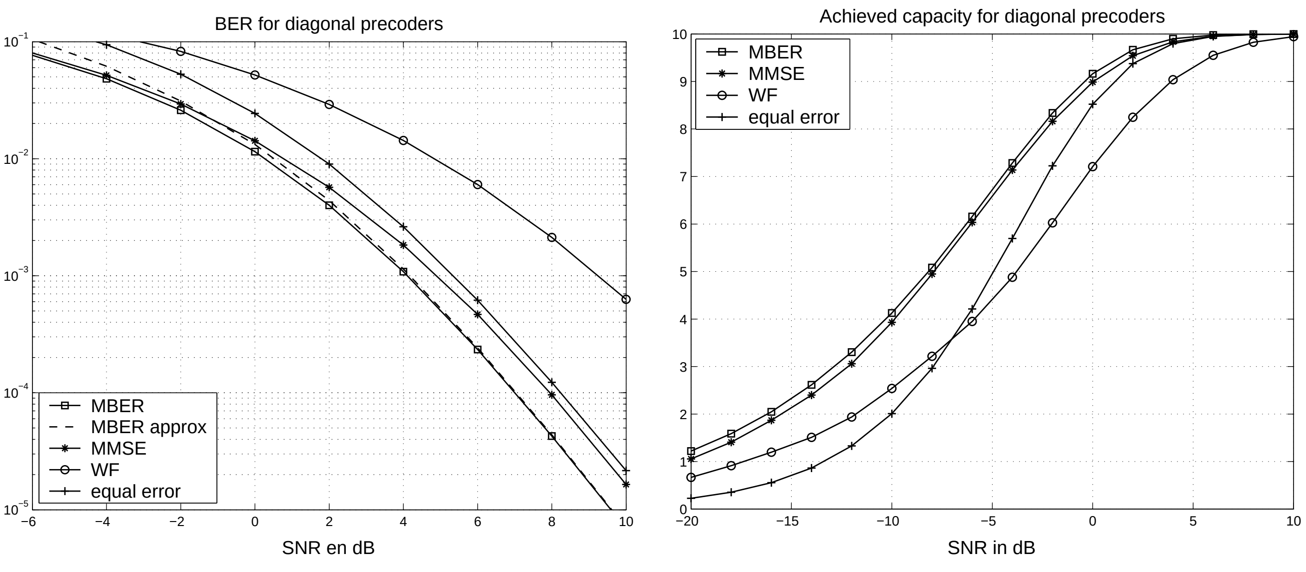
The efficiency of the MBER precoder is illustrated by Fig. 2 which represents the BER and achieved capacity of MIMO systems with different precoders with respect to the SNR. We here consider $n_T = 5$ transmitters, $n_R = 5$ receivers; using a QPSK constellation, we transmit 4 independent data streams over the system.

For each SNR, $100\,000$ 4-symbol vectors are transmitted during the Monte-Carlo simulations. For each transmitted vector, a new H and a new R are randomly chosen in order to obtain results that depend neither on a particular channel, nor on particular noise statistics. Entries of H are i.i.d. zero-mean unit-variance complex Gaussian random variables. Matrices R are obtained by $R = TT^*$ (where entries of T are i.i.d. zero-mean unit-variance complex Gaussian random variables) and then scaled according to the desired SNR. The SNR is defined as the ratio of the total transmitted power p_0 to the total received noise power.

In order to compute the achieved capacity, we consider a binary symmetric channel (BSC) [6] because we use an optimal receiver (ML) and the input and the output of the system can be seen as binary data streams. In the MIMO case with b independent data streams and a QPSK constellation (2 bits per symbol), the BER and the achieved capacity are then related by the following expression:

$$C_{QPSK} = 2b \times (1 + BER \times \log_2(BER) + (1 - BER) \times \log_2(1 - BER)) \quad \text{(bits / transmitted vector)} \quad (17)$$

Fig. 2 shows that the best precoder is the MBER, followed by the MMSE. In the MMSE solution, the performances of our equivalent model are exactly the same as in [9]. In opposition with the expected capacity, the WF is the worst one (the SNR gain of the MBER is about 5 dB). Intermediate results are obtained by the equal-error precoder. Finally, the performances of the very simple AMBER are close to the optimal MBER and even equivalent as the SNR is increasing.



(a) Bit-error-rate

(b) Achieved capacities (bits/s/Hz)

Figure 2: Experimental results for a QPSK with $n_T = 5$, $n_R = 5$, and $b = 4$

V CONCLUSION

In this paper, we proposed a simplified representation of MIMO channels along with an optimization of the global transmission scheme. Thanks to simple matrices manipulations, *i.e.* an eigen value decomposition and a singular value decomposition, we respectively whitened the noise and diagonalized the channel. The diagonality of the obtained global system allowed us to design a diagonal precoder based on the Minimum Bit Error Rate criterion. By comparisons precoders based on other criteria, such as the maximum information rate or the minimum mean square error, it was shown that the MBER precoder was far more efficient, both in achieved capacity and BER terms. The best compromise between performance and complexity is certainly obtained by the AMBER precoder. Its performances are very close to the optimal MBER one with a minimum of complexity.

The diagonality the global optimized system allows independent parallel subchannels and an easy ML reception. Nevertheless, it would be interesting to study the design of non-diagonal precoders and their performance under the same conditions. Futhermore, this kind of transmission scheme is directly applicable to narrowband wireless channels, in which channel information can be made available at the transmitter. As a matter of fact, future research will include applications to broadband MIMO channels, with the help of multicarrier modulations.

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