DIGITAL TRANSMISSIONS WITH CHAOTIC SIGNALS: FAST RECEIVER SYNCHRONIZATION USING DUPLICATED CHAOTIC OSCILLATORS

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ABSTRACT

In recent years, synchronization of chaotic systems and its potential application to secure digital communications have received ever increasing attention.

Due to their high sensitivity to initial conditions, chaotic oscillators are difficult to synchronize. Furthermore, synchronization must be performed for each new symbol, so synchronization time puts a strong limitation on possible bit rates (the symbol period cannot be less than the synchronization time). In this paper, we propose an improvement to the structure of a chaotic receiver, in order to reduce the time needed for synchronization. The idea is to duplicate each receiver chaotic oscillator. A free-mode version of each receiver oscillator is used as a short term dynamic memory. At the end of each symbol period, information is transferred between the copies and the initial oscillators.

Experimental results show that using this method yields to a significant reduction of synchronization time.

1. INTRODUCTION

In a conventional digital communication system, bit sequences are mapped to sinusoids, or sums of sinusoids. In a chaotic communication system, chaotic waveforms are used instead of sinusoids. Research in chaotic systems is motivated by its potential interest for secure communications. Indeed, a chaotic signal is very difficult to demodulate for someone who does not know the parameters of the transmitter chaotic oscillators.

A chaotic oscillator is a system which is extremely dependent on the initial conditions [1]. If we consider two identical chaotic oscillators, an extremely small difference of their initial state causes the signals they generate to quickly diverge. A chaotic signal is therefore unpredictable in the long term.

Synchronization is the hard point in a chaotic digital communication system [3]. Indeed, the feasibility of chaotic oscillators synchronization is not obvious, because the characteristic of chaotic oscillators is sensitive dependence to initial conditions. However, it has been proved that synchronization is possible if the chaotic oscillator is modified in order to evolve in a driven-mode: an error signal, based on the received signal, is injected in the oscillator equations [4][5].

In this paper, we propose a contribution to synchronization of a chaotic receiver, that reduces the time required for convergence of synchronization. The idea is to duplicate each chaotic oscillator, one copy evolves in a driven-mode, while the other evolves in a free-mode. The latter is used as a short term dynamic memory.

The paper is organized as follows: in Section 2, the chaotic oscillator we use is briefly described. In Section 3, we show how it can be synchronized. Section 4 presents a chaotic communication system: we show that the long synchronization time causes reception errors if the symbol period is too small. In Section 5, we study the speed of divergence of a chaotic oscillator. Using this result, we can say that a free-mode chaotic oscillator can be used as a short term dynamic memory if the symbol period is less than its divergence time. This idea is used in Section 6, where we present our approach. Experimental results show that the method is able to considerably reduce the synchronization time.

2. CHUA'S CHAOTIC OSCILLATOR

Chua's circuit [2][4][6] is shown on figure 1. It contains a nonlinear negative resistance, which provides energy and is essential to obtain a chaotic behavior.

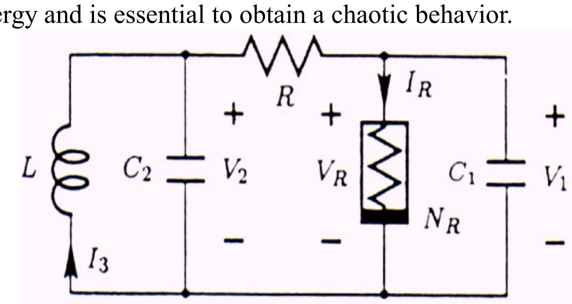

Figure 1 : Chua's circuit

The circuit can be realized, or simulated using differential equations (where $G = 1 / R$):

$$\frac{dV_1}{dt} = \frac{G}{C_1}(V_2 - V_1) - \frac{1}{C_1} f(V_1) \qquad (1)$$

$$\frac{dV_2}{dt} = \frac{G}{C_2}(V_1 - V_2) + \frac{1}{C_2} I_3 \qquad (2)$$

$$\frac{dI_3}{dt} = -\frac{1}{L} V_2 \qquad (3)$$

$$f(V_1) = G_b V_1 + \frac{1}{2}(G_a - G_b)(|V_1 + E| - |V_1 - E|) \qquad (4)$$

In the sequel, we will use the same parameters as in [5]:
$L = 18$ mH, $C_1 = 10$ nF, $C_2 = 100$ nF, $R = 1800$ Ohms,
$G_a = -50 / 66$ mS, $G_b = -9 / 22$ mS, $E = 1$ V.

Figures 2 and 3 show respectively an example of the oscillator output signal (V_1 as a function of time), and V_2 as a function of V_1. Figure 3 can be used for theoretical study of the oscillator. It clearly shows two attractors.

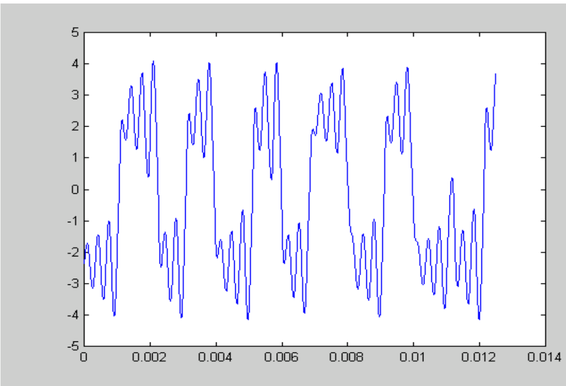

Figure 2 : V_1 with respect to time

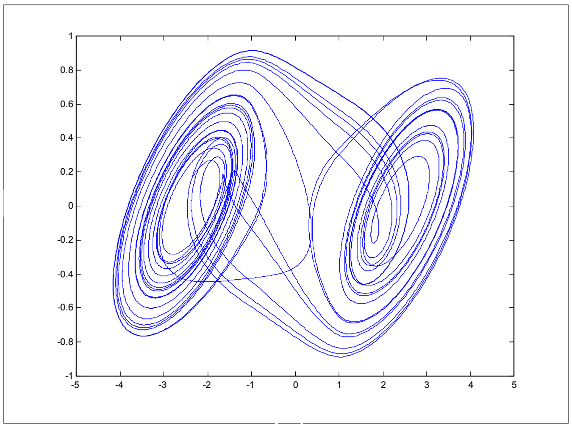

Figure 3 : V_2 with respect to V_1

3. SYNCHRONIZATION OF A CHAOTIC OSCILLATOR

A chaotic signal $V_1(t)$ is generated by a chaotic oscillator, and transmitted over a channel which adds a gaussian and centered white noise. The channel may be subject to interruptions of transmission. The receiver has the same oscillator as the transmitter, and must synchronize it, using the received signal *r(t)* only.

Synchronization is performed by computing an error signal *e(t),* which is the difference between the received signal *r(t)* and the estimated transmitted signal $\hat{V}_1(t)$. This error is injected in the circuit. The circuit equations in driven-mode are:

$$\frac{d\hat{V}_1}{dt} = \frac{G}{C_1}(\hat{V}_2 - \hat{V}_1) - \frac{1}{C_1} f(\hat{V}_1) + \frac{G_E}{C_1} e(t) \qquad (5)$$

$$\frac{d\hat{V}_2}{dt} = \frac{G}{C_2}(r(t) - \hat{V}_2) + \frac{1}{C_2} \hat{I}_3 \qquad (6)$$

$$\frac{d\hat{I}_3}{dt} = -\frac{1}{L} \hat{V}_2 \qquad (7)$$

$$f(\hat{V}_1) = G_b \hat{V}_1 + \frac{1}{2}(G_a - G_b)(|\hat{V}_1 + E| - |\hat{V}_1 - E|) \qquad (8)$$

$$e(t) = r(t) - \hat{V}_1(t) \qquad (9)$$

In our experiments, we used the following value for the coupling resistance: $R_E = 1 / G_E = 30000$ Ohms.

The transmitted signal is shown on figure 2. Figure 4 shows the received signal: it is noisy and interrupted. Figure 5 shows the output of the receiver chaotic oscillator (i.e. estimated transmitted signal). Figure 6 shows the error signal (difference between figures 5 and 2).

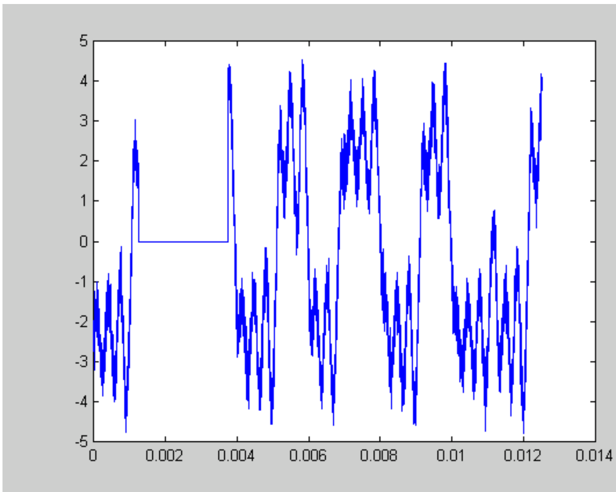

Figure 4: Received signal $r(t)$

We can note that the transmission interruption causes the receiver to progressively desynchronize (the error increases: see fig. 6). Then, when transmission is restored, the error decreases: this means that the receiver resynchronizes. We can also note that the noise is almost totally suppressed by the receiver oscillator (figure 5)

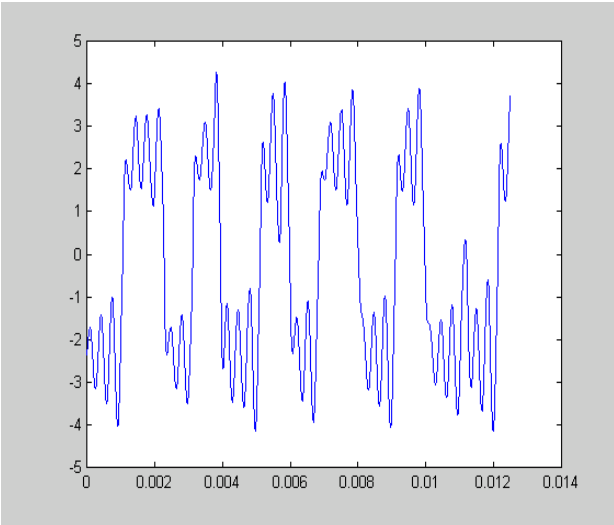

Figure 5: Estimated transmitted signal

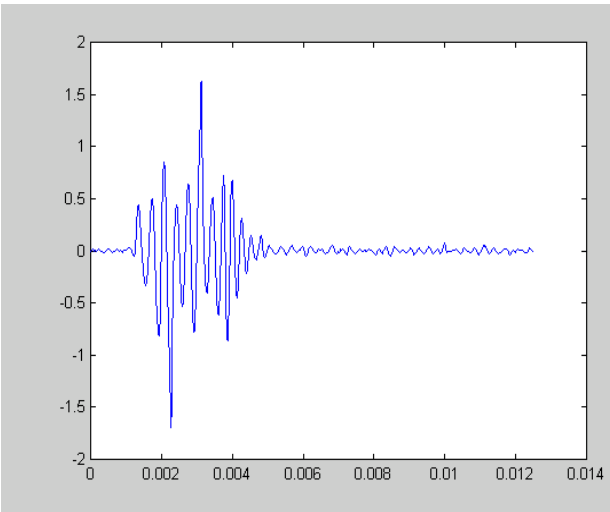

Figure 6 : Error signal $e(t)$

4. APPLICATION TO DIGITAL COMMUNICATIONS

Let us consider a digital transmission with binary symbols. On the transmitter side, we use two chaotic oscillators:

- Oscillator A was defined in section 2
- Oscillator B is also a Chua's circuit, but the parameters are:
  $L = 20$ mH, $C_1 = 10$ nF, $C_2 = 80$ nF, $R = 1960$ Ohms
  $G_a = -50 / 66$ mS, $G_b = -9 / 22$ mS, $E = 1.5$ V

The transmitted signal is built as follows: time is divided into windows of equal duration. Each window is used to transmit one symbol. When the symbol is 0, the window contains the signal from oscillator A, and when the symbol is 1, it contains the signal from oscillator B.

On the receiver side, we use the same oscillators, but in driven-mode. The decisions are made by a correlator: the received signal is correlated with the driven-mode oscillators output. If oscillator A (resp. B) provides the highest correlation, the estimated transmitted symbol is 0 (resp. 1).

Let us consider the transmission of the binary sequence: 1 0 1 0 1. Figures 7 and 8 show the output signal of transmitter oscillators A and B, and figure 9 shows the received signal.

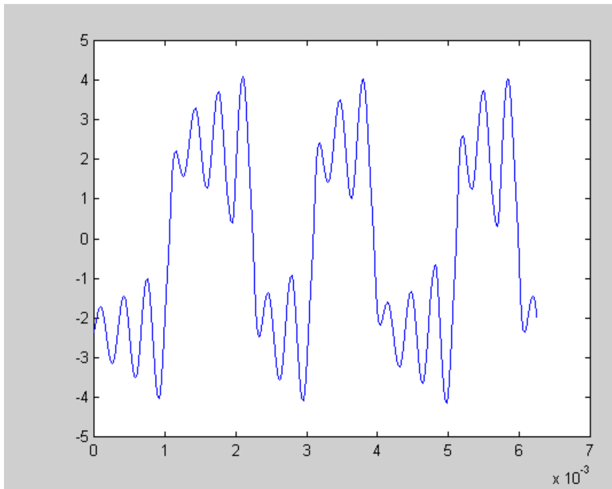

Figure 7: Output signal of transmitter oscillator A

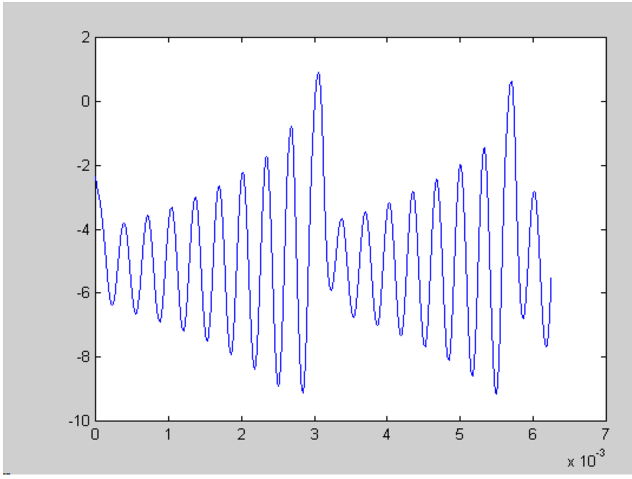

Figure 8: Output signal of transmitter oscillator B

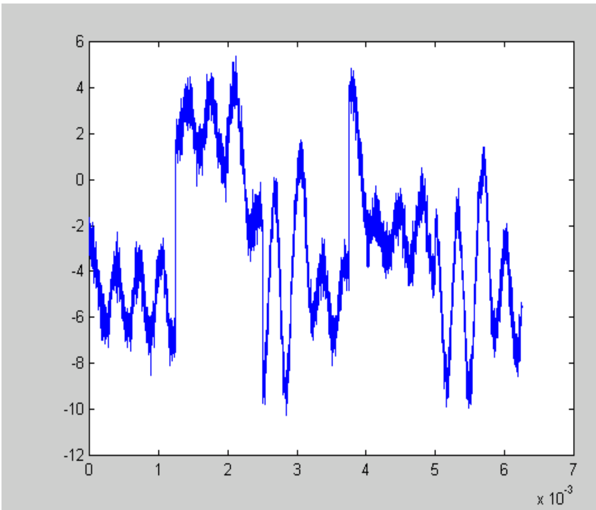

Figure 9: Received signal

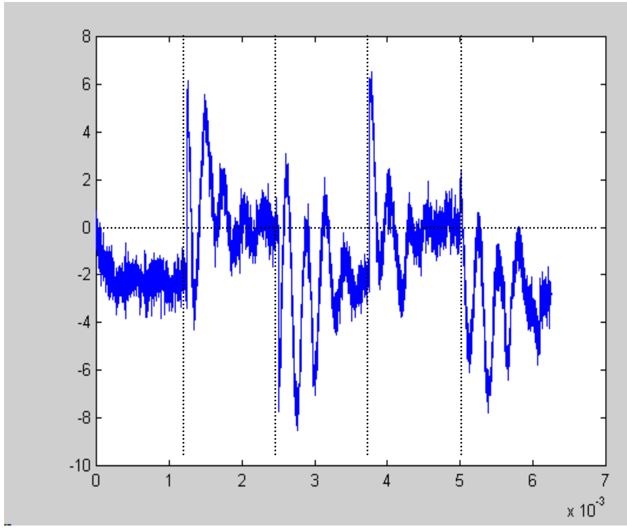

Figure 10: Error signal $e(t) = r(t) - \hat{V}_1(t)$ **for receiver oscillator A**

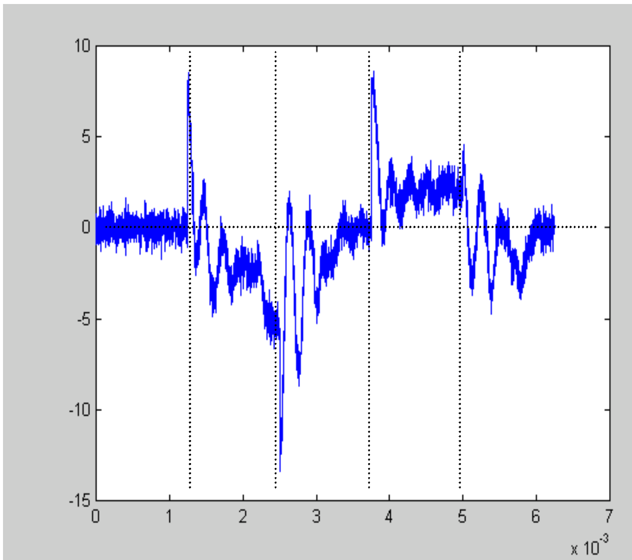

Figure 11: Error signal $e(t) = r(t) - \hat{V}_1(t)$ **for receiver oscillator B**

Figures 10 and 11 show the error signals of receiver oscillators A and B. Vertical lines have been added manually on figures 10 and 11 to show (approximately) the limits of the time windows. For instance, let us consider the third time window. The transmitted bit is 1, hence the transmitted signal comes from oscillator B. We can see that the receiver oscillator B synchronizes (the error decreases), while oscillator A cannot synchronize. Hence, the output signal of receiver oscillator B will be the closest to the received signal and the decision will be correct. However, we can see that oscillator B requires approximately two thirds of the symbol period to synchronize. Hence, it is not possible to increase the bit rate without causing reception errors.

5. SPEED OF DIVERGENCE OF A CHAOTIC OSCILLATOR

Let us consider two identical chaotic oscillators of type A. Even when they start from extremely close initial conditions, their output signals quickly diverge. Figure 12 show the natural logarithm of the absolute value of their output signals difference, with respect to time.

This figure clearly shows that the oscillators diverge exponentially until t=0.006. At that time, their output signals have become totally independent.

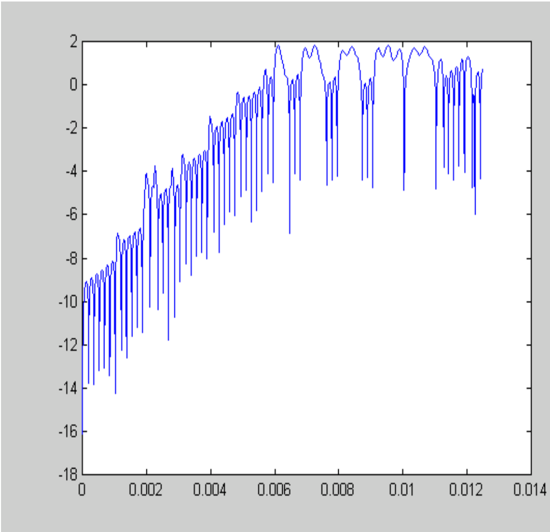

Figure 12: Natural logarithm of the absolute value of the difference between chaotic oscillators output signals, with respect to time.

6. FAST SYNCHRONIZATION

6.1 Principle

Let us consider the third time period again (figure 11). At the end of the first time period, oscillator B was synchronized because the transmitted bit was 1. But it was desynchronized during the second time period because the transmitted bit was 0. Hence, in the third time period, it requires time to synchronize again. In fact, each time a 0 is transmitted, oscillator B desynchronizes, and it needs time to resynchronize when a 1 is transmitted.

Now, if we could imagine a way to partially preserve synchronization during the second time period, it is clear that the synchronization phase during the third time period would be considerably shorter.

The time period is approximately 1.2×10^{-3}. Figure 12 shows that the time required to fully desynchronize a free-mode oscillator is about 5 times larger. Hence, we propose to use free-mode oscillators as short term dynamic memories in the receiver. Figure 13 shows the proposed approach. At the end of each time period:

- The state of the best synchronized driven-mode oscillator is loaded **into** its free-mode copy. Hence, if the driven-mode oscillator desynchronizes during the next time period, its free-mode copy will remain almost synchronized.
- The state of the other oscillator is loaded **from** its free-mode copy.

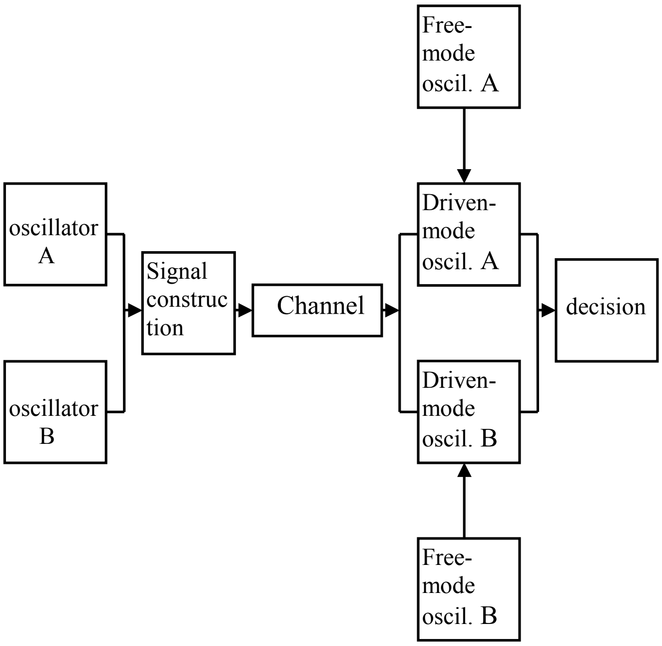

Figure 13 : Proposed method

One could object that, if a long sequence of 0 is transmitted, the free-mode oscillator B will also desynchronize. That is true, but 2 remarks can be done:

1. The probability of a long sequence of 0 is low.
2. There exist bit stream coding strategies that avoid long sequences of 0.

6.2 Experimental results

The bit stream is the same as in Section 4. Figures 14 and 15 show the error signals of driven-mode receiver oscillators A and B. These figures are to be compared with figures 10 and 11. Here, we can see that the synchronization time of the driven-mode oscillator which corresponds to the transmitted bit is always very fast. Hence, the symbol period could be reduced (i.e. the bit rate could be increased).

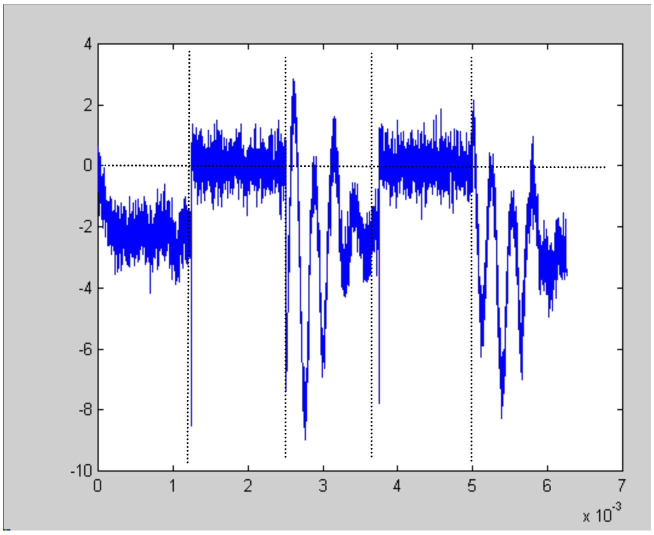

Figure 14: Error signal for receiver driven-mode oscillator A

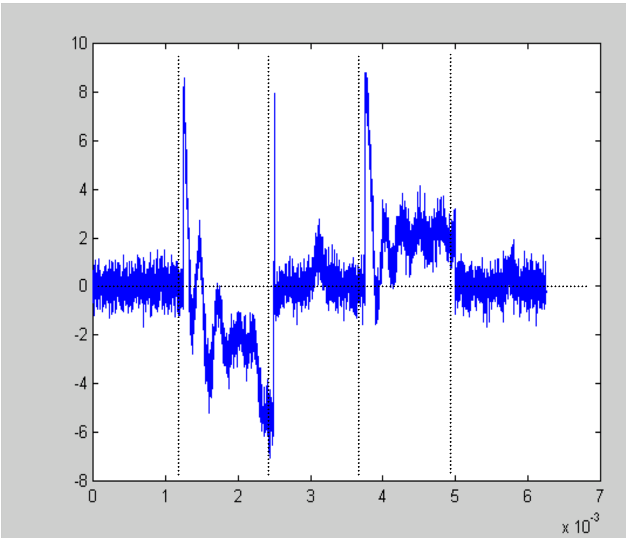

Figure 15: Error signal for receiver driven-mode oscillator B

7. CONCLUSION

We have proposed a method to increase the synchronization speed of chaotic oscillators in a digital transmission context. The method is based on transfer of information between driven-mode and free-mode oscillators, the latter being used as dynamic short term memories.

There is no doubt that chaotic oscillators are promising systems for secure digital communications. However, research in this area is still young, and the robustness of these methods in difficult environments (noise, echoes) still remains to be improved.

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