A robust synchronization procedure for blind estimation of the symbol period and the timing offset in spread spectrum transmissions

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Abstract - **In the context of spectrum surveillance (non cooperative context), a robust synchronization procedure is presented, in order to estimate the period symbol and the timing offset of a direct sequence spread spectrum signal. Experimental results are given to illustrate the performances of the algorithm.**

I Introduction

Spread spectrum transmissions have been in practical use since the 1950's [1]. They found many applications in military systems due to their low probability of interception. Nowadays, they are also widely used for commercial applications, especially for code division multiple access (CDMA) or global positioning system (GPS) [2].

Direct-sequence spread spectrum (DS-SS) is a transmission technique in which a pseudo-noise sequence or pseudo-random code [3], independent of the information data, is employed as a modulation waveform to spread the signal energy over a bandwidth much larger than the information signal bandwidth [4]. Indeed, the DS-SS signal can be transmitted below the noise level. In practical systems, the pseudo-random sequence, as well as the carrier and symbol frequencies, are known by the receiver. The signal is then correlated with a synchronized replica of the pseudo-noise code at the receiver side, in order to retrieve the symbols. However, in the context of spectrum surveillance, all these parameters are unknown. It becomes therefore very difficult to detect and demodulate a DS-SS signal. In this context, Tsatsanis *et al.* have proposed a reliable method to recover the convolution of the PN sequence and the channel response in multipath environment [5], assuming that the chip period T_c is known with high precision, as well as the number of symbols in the PN sequence.

In [6], we proposed a simple efficient algorithm to estimate the spreading sequence, assuming only a precise estimation of the symbol period T. However, synchronization is the weakest point of this algorithm. Indeed, it is a difficult problem because synchronization must be performed in a blind context, that is without knowing the spreading sequence, and even before estimating it. This is the reason why, in this paper, we propose a more robust blind synchronization algorithm. We only assume the signal has been detected and the symbol period has been estimated through the method proposed in [7]. We remind that the procedure is based on two parallel computations : the "theoretical path", in which we compute the theoretical behavior of the fluctuations of the second order moments estimators in the case noise alone is present, and the "experimental path", in which the actual fluctuations are computed. When a DS-SS signal is hidden in the noise, the actual fluctuations go outside the noise-only bounds provided by the theoretical path for every delay multiple of the symbol period. We are going to show that the synchronization approach enables us to adjust more precisely the symbol period estimation.

The paper is organized as follows. In section 2, we give the notations and hypotheses. Then the proposed approach is described in section 3. Finally, numerical results are provided to illustrate the method in section 4 and a conclusion is drawn in section 5.

II Problem formulation

A *Notations and hypotheses*

In a DS-SS transmission, the symbols a_k, whose constellation is usually a quaternary phase shift keying (QPSK), are multiplied by a PN code $\{c_k\}_{k=0\ldots P-1}$ of chip duration T_c, which spreads the bandwidth. This signal is then filtered, sent through the channel and filtered again at the receiver side; The convolution of all the filters of the transmission chain will be denoted as $p(t)$. The resulting baseband signal is given by

$$y(t) = \sum_{k=-\infty}^{+\infty} a_k h(t-kT) + n(t) \qquad (1)$$

with

$$h(t) = \sum_{k=0}^{P-1} c_k p(t-kT_c) \qquad (2)$$

where $n(t)$ stands for the noise at the output of the receiver filter. The hypotheses below will be assumed :
1. The symbol period T has been estimated according to [7] ; All other parameters are unknown;
2. The noise is averaged white gaussian (AWG) and uncorrelated with the symbols (centered and uncorrelated);
3. The signal to noise ratio (SNR (dB)) at the output of the receiver filter is negative: the signal is hidden in the noise.

B *Problem analysis*

The received signal is sampled and divided into non overlapping windows, the duration of which is T. Let us note $\mathbf{x}$ the content of a window, and define the correlation matrix $\mathbf{R} = E\left\{\mathbf{x}.\mathbf{x}^H\right\}$, where ^{H} denotes the Hermitian transpose. In [6] it has been proved, that the eigenanalysis of this matrix yields two large eigenvalues :

$$\lambda_1 = \left(1+\rho\frac{T-t_0}{T_e}\right)\sigma_n^2 \ , \ \lambda_2 = \left(1+\rho\frac{t_0}{T_e}\right)\sigma_n^2 \quad (3)$$

where ρ denotes the SNR, t_0 is the unknown desynchronization time between the symbols and the window, T_e is the sampling period and $\sigma_n^2 = \lambda_i \ \forall \ i \geqslant 3$ is the noise variance.

In [6], we have proved, that the spreading sequence can be recovered from the first and the second eigenvector corresponding to the two large eigenvalues. However, the desynchronization time must be estimated to know which eigenvector describes the beginning of the sequence and which one the end. The desynchronization time t_0, as well as the SNR ρ, can be estimated from (3):

$$\begin{cases} \widehat{\rho} = \left(\frac{\lambda_1+\lambda_2}{\sigma_n^2} - 2\right)\frac{T_e}{T} \\ \widehat{t_0} = \frac{T_e}{\widehat{\rho}}\left(\frac{\lambda_2}{\sigma_n^2} - 1\right) \end{cases} \quad (4)$$

We have also noted that the less accurate the value of $\widehat{t_0}$ will be, the higher the noise variance will be. Therefore we perform the desynchronization time estimation $\widehat{t_0}$ by using the more robust algorithm proposed below, based on the relation between the correlation matrix and its eigenvalues.

III Robust Synchronization of DS-SS Signals

In a first part, we give notations and present the outline of our synchronization procedure. Then we will focus on implementation aspects.

A *Overview of the synchronization scheme*

Let $\mathbf{y}$, a column vector, be the discrete version of the received baseband signal $y(t)$:

$$\mathbf{y} = \{y(k)\}_{k=1,2,\ldots} = \{y((k-1)Te)\}_{k=1,2,\ldots} \quad (5)$$

where $T_e = 1/F_e$ denotes the sampling period. Throughout the sequel, the knowledge of a rough estimation $T^{(0)}$ of the symbol period, according to [7], is assumed before running our algorithm. Then, the problem under consideration is to "fine tune" the symbol period estimation around its initial value $T^{(0)}$ and to decide which sample in $\mathbf{y}$ corresponds to the beginning of the first symbol. We will proceed by iterations to address these closely related problems. The best estimate of the symbol period will be searched in the following set

$$\mathcal{T}_{\varepsilon_s}^{\bar{n}} = \left\{T_n \in \mathbb{R} : T_n = T^{(0)}(1+n\varepsilon_s) \ , \ n \in [-\bar{n}, +\bar{n}]\right\} \quad (6)$$

Notice that $\mathcal{T}_{\varepsilon_s}^{\bar{n}}$ is of very limited dimension (less than 20, typically) if a good initial estimate $T^{(0)}$ is considered, which is true if the detector [7] has been used. The beginning of the first symbol will be searched in $\mathbf{y}$ for all the elements of $\mathcal{T}_{\varepsilon_s}^{\bar{n}}$, as explained in the sequel.

First, the signal is resampled according to a particular value T_n of the symbol duration, in order to have an integer number of samples per symbol. M will denote the number of samples per symbol and $(N+1)$ will be the total number of symbols in $\mathbf{y}$.

Then, the beginning of the first symbol $\mathbf{y}(\hat{d}+1)$ is searched in a set of M samples :

$$\hat{d} \in [0, M-1] \quad (7)$$

The desired estimate $\hat{d}$ will be derived from a set of covariance matrices $\{\mathbf{R}_d\,(T_n)\}_{d\in[0,M-1]}$ corresponding to the set of time-domain analysis window $\{\mathbf{y}_{d,k};\ k = 1, 2, ..., N\}_{d\in[0,M-1]}$:

$$\mathbf{y}_{d,k} = \mathbf{y}(d+(k-1)\ M+1 : d+k\ M) \quad (8)$$

$$\mathbf{R}_d = \frac{1}{N}\sum_{k=1}^{N}\mathbf{y}_{d,k}.\mathbf{y}_{d,k}^H \simeq E\left\{\mathbf{y}_{d,k}.\mathbf{y}_{d,k}^H\right\} \quad (9)$$

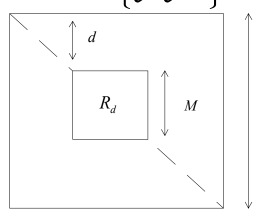
where the subscripting notation $\mathbf{v}(i:j)$ stands for the sub-vector with elements $i \cdots j$ of vector $\mathbf{v}$.

Now, a signal matrix of augmented size (2M rows, N columns) is constructed as:

$$\mathcal{Y} = \left[\begin{array}{cccc} \mathbf{y}_{0,1} & \mathbf{y}_{0,2} & \cdots & \mathbf{y}_{0,N} \\ \hline \mathbf{y}_{0,2} & \mathbf{y}_{0,3} & \cdots & \mathbf{y}_{0,N+1} \end{array}\right] \quad (10)$$

Then, it is obvious that the associated covariance matrix yields the whole set $\{\mathbf{R}_d\,(T_n)\}_{d\in[0,M-1]}$:

$$\mathcal{R} = \left\{ \quad E\left\{\mathcal{Y}.\mathcal{Y}^H\right\} \right. \quad (11)$$



Now, it is shown in the following property that $\|\mathbf{R}_d\|$ contains information about the desynchronization time, where $\|.\|$ denotes the Frobenius matrix norm :

Property 1 *: The beginning of the searched symbol can be obtained as the following maximum :*

$$\hat{d} = \arg\max_{d\in[0,M-1]}\left\{\|\mathbf{R}_d\,(T_n)\|^2\right\} \quad (12)$$

Obviously, the estimated desynchronization time is $\widehat{t_0} = \hat{d}.T_e$.

Proof. : From equations (4), it appears that λ_1 is maximum when the desynchronization is null. Hence, a good approach to estimate d is to maximize the first eigenvalue.

However, this approach is computationally intensive because synchronization has to be performed for each element of $\mathcal{T}_{\varepsilon_s}^{\bar{n}}$. Thus, this approach would require many eigenvalue computations. This is the reason why we propose an alternative approach, which provides the same result, but without requiring explicit computation of the eigenvalues.

From basic eigenanalysis theory [8], we know that:

$$\|\mathbf{R}_d\|^2 = \sum_{i=1}^{N} (\lambda_i)^2 \tag{13}$$

Since $\{\lambda_i,\, i \geq 3\}$ does not depend on the desynchronization time (3), it is clear that $\|\mathbf{R}_d\|^2$ is maximum when $(\lambda_1)^2 + (\lambda_2)^2$ is maximum. Using (3), we can write:

$$\lambda_1 + \lambda_2 = c \tag{14}$$

where c is a constant which does not depend on the desynchronization time. Hence:

$$(\lambda_1)^2 + (\lambda_2)^2 = 2\,(\lambda_1)^2 - 2c\lambda_1 + c^2 \tag{15}$$

The derivative of this expression with respect to λ_1 is $4\lambda_1 - 2c$. Let us assume that the eigenvalues are sorted in decreasing order. The derivative above is always positive, because $\lambda_1 \geq c/2$ (otherwise, (14) would lead to $\lambda_2 > \lambda_1$). The result is that $\|\mathbf{R}_d\|^2$ is an increasing function of λ_1, hence maximizing λ_1 is equivalent to maximizing $\|\mathbf{R}_d\|^2$.

The interest of this approach is that computation of $\|\mathbf{R}_d\|^2$ is considerably faster than computation of the eigenvalues (furthermore, we can take advantage of redundancies between matrices $\mathbf{R}_d$ for successive values of d). ■

This iterative process will be repeated for each value of the symbol duration around its first estimate $T^{(0)}$, and the final synchronization will be achieved as

$$(\hat{T}, \hat{d}) = \arg \max_{\substack{d \in [0, M-1] \\ T \in \mathcal{T}_{\varepsilon_s}^{\bar{n}}}} \left(\|\mathbf{R}_d(T)\|^2 \right) \tag{16}$$

Then, the signal is delayed according to this last estimate :

$$\widetilde{\mathbf{y}} = \left[\mathbf{y}_{\hat{d},1}^H,\ \mathbf{y}_{\hat{d},2}^H,\ \ldots,\ \mathbf{y}_{\hat{d},N}^H \right]^H \tag{17}$$

This synchronized version of the received signal is then used to estimate the spreading sequence, using the method described in [6].

B *The synchronization algorithm*

Numerous simulations have revealed that our spreading sequence estimation procedure performs well for short data records (duration of $y(t)$), even at low signal to noise ratio ($SNR \sim -10\ dB$). Hence, the received discrete-time signal $\mathbf{y}$ is first truncated to keep $(N+1) \sim 200$ symbols (signal time duration $D = (N+1).T^{(0)}$) in accordance with the initial sampling frequency and the initial symbol duration $T^{(0)}$. A resampling by linear interpolation of $y(t)$ is then done (frequency F_e, sampling period T_e) to have an integer number of samples per symbol. In order to guarantee the robustness of the synchronization, the set $\mathcal{T}_{\varepsilon_s}^{\bar{n}}$ in (6) is constructed with a proper value of ε_s :

$$\varepsilon_s = \frac{T_e}{2D} \tag{18}$$

Hence, the synchronization process will lead to an error of $1/4$ chip in the worst case.

In order to limit the running time of the algorithm, we have found that it is judicious to apply the estimation procedure in two steps: we begin with a medium accuracy ε_s^-, and then the final high accuracy ε_s is considered (18). The main steps involved in our synchronization procedure are then summarized as follows.

Step 0) We make the assumption that a "rough estimation" of the symbol duration is available : $T^{(0)}$

Step 1) The following discrete set is then constructed:

$$\mathcal{T}_{\varepsilon_s^-}^{\bar{n}^-} = \left\{ T_n \in \mathbb{R} : T_n = T^{(0)}(1 + n\varepsilon_s^-)\ ,\ n \in \left[-\bar{n}^-, +\bar{n}^-\right] \right\}$$

where the time sampling parameter is given as (for example)

$$\varepsilon_s^- = 4\varepsilon_s$$

Usually, $\bar{n}^- = 5$ is seen to be a sufficient value. Proceeding by iterations, all the values of symbol duration $T_n \in \mathcal{T}_{\varepsilon_s^-}^{\bar{n}^-}$ will be considered to process as follows:

1. A resampling step is performed in order to have an integer number of samples per symbol. M will denote the number of samples per symbol, and $N+1$ will be the total number of symbols.
2. The partitioned signal matrix $\mathcal{Y}$ is constructed as in (10).
3. The covariance $\mathcal{R}$ associated to $\mathcal{Y}$ is computed. As shown in (11), a set of covariance matrices $\{\mathbf{R}_d\,(T_n)\}_{d \in [0, M-1]}$ corresponding to various delays d of analysis windows is readily obtained as diagonal blocks.
4. The best estimate of the beginning of the first symbol is derived as (12).

The best synchronization with accuracy ε_s^- will be achieved in $(2\bar{n}^- + 1)$ iterations as :

$$(T^{(1)}, d^{(1)}) = \arg \max_{\substack{d \in [0, M-1] \\ T \in \mathcal{T}_{\varepsilon_s^-}^{\bar{n}^-}}} \left(\|\mathbf{R}_d(T)\|^2 \right) \tag{19}$$

Step 2) A second discrete set is constructed for the symbol duration:

$$\mathcal{T}_{\varepsilon_s}^{\bar{n}} = \left\{ T_n \in \mathbb{R} : T_n = T^{(1)}(1 + n\varepsilon_s)\ ,\ n \in [-\bar{n}, +\bar{n}] \right\}$$

where $\bar{n} = 3$, typically. Then, the iterative process described before (from 1. to 4.) is repeated and the final synchronization will be obtained as:

$$(\hat{T}, \hat{d}) = \arg \max_{\substack{d \in [0, M-1] \\ T \in \mathcal{T}_{\varepsilon_s}^{\bar{n}}}} \left(\|\mathbf{R}_d(T)\|^2 \right) \tag{20}$$

IV Simulations

To describe the approach, a DS-SS signal is generated using a complex random sequence of length 31 with a chip period $T_c = 0.02\ \mu s$. The symbols belong to a QPSK constellation, which period is $T = 0.62\ \mu s$. We assume the first step in our blind spreading sequence estimation procedure has still to be done using the detector [7]. We have then an estimation $T^{(0)}$ of the symbol period, ($T^{(0)} = 0.6196\mu s$), which we have now to adjust to be well synchronized. The SNR at the output of the receiver filter is equal to -10 dB.

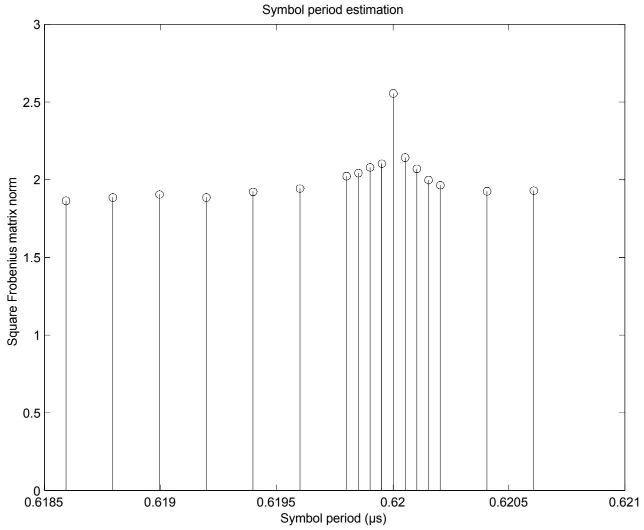


Fig. 1. Symbol period estimation

In Fig. 1 the values of the criterion for all the estimated symbol durations T_n are shown. The central part of the figure represents step 2 of the algorithm with the proper value of ε_s, while step 1 is illustrated on each side with $\varepsilon_s^- = 4\varepsilon_s$. It is clearly seen on this figure, that the best symbol period estimation is $\widehat{T} = 0.62\ \mu s$.

Point 4 of the algorithm is represented on Fig. 2, where the squared Frobenius matrix norm is drawn versus the delay d. The best estimate of the beginning of the first symbol is equal in this example to $\widehat{t_0} = 0.10\ \mu s$.

Once we have answered the two questions above, the signal is delayed according to (17). As described in section II. B, we can define the correlation matrix $\mathbf{R} = \mathbf{R}_{\widehat{d}}(\widehat{T})$.

Since the signal is well synchronized, the eigenanalysis of $\mathbf{R}$ shows that there is only one large eigenvalue, whose corresponding eigenvector provides the spreading sequence [6].

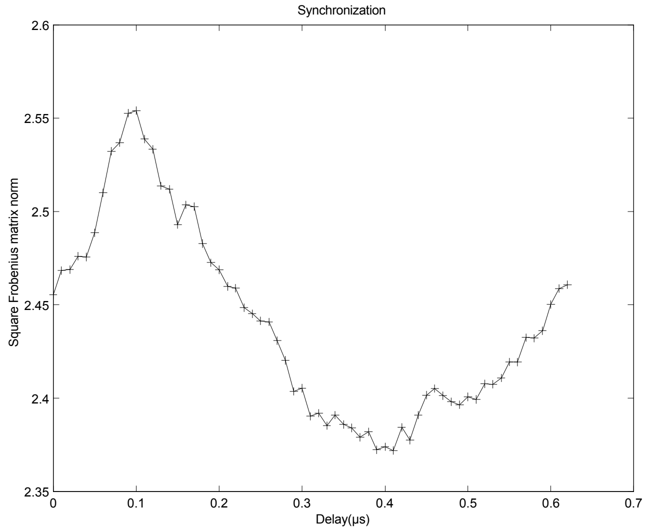


Fig. 2. Synchronization

V Conclusions

In this paper, the problem of DS-SS signals synchronization has been considered in a context of blind estimation. Indeed, in spectrum surveillance applications, we do not know the spreading sequence. Our procedure is based on an iterative search of the beginning of the first symbol for various symbol durations in the neighborhood of an initial estimate. The property that an efficient synchronization can be achieved by maximizing the squared Frobenius norm of a covariance matrix is of central importance in our approach. We emphasize the limited computational cost of the algorithm by a proper choice of the initial symbol duration, using results from [7]. A numerical simulation illustrates the robustness of the method in a very noisy environment (SNR of -10 dB).

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