

Sweeping processes and Stieltjes derivative

Bianca Satco

Stefan cel Mare University of Suceava (Romania)

Abstract.

Let $g : [0, T] \rightarrow \mathbb{R}$ be a right-continuous nondecreasing function and μ_g the Lebesgue-Stieltjes measure generated by g .

The aim of this talk is to present an existence result ([3]) for a perturbed differential inclusion governed by a sweeping process with state dependent convex moving set

$$\begin{cases} -u'_g(t) \in N_{C(t, u(t))}(u(t)) + F(t, u(t)), \mu_g - a.e. t \in (0, T) \\ u(0) = u_0 \in C(0, u_0). \end{cases}$$

The moving set $C : [0, T] \times \mathbb{R}^d \rightarrow \mathcal{P}_{kc}(\mathbb{R}^d)$ satisfies the condition

$H)$ there exist $L_1 \geq 0, L_2 \in [0, 1)$ such that for every $s, t \in [0, T]$ and $u, v \in \mathbb{R}^d$:

$$D(C(t, u), C(s, v)) \leq L_1 |g(t) - g(s)| + L_2 |u - v|$$

while the set-valued perturbation $F : [0, T] \times \mathbb{R}^d \rightarrow \mathcal{P}_{kc}(\mathbb{R}^d)$ is a Carathéodory mapping subject to a usual growth assumption.

The novelty brought by our study, namely the use of the Stieltjes derivative u'_g with respect to $g : [0, T] \rightarrow \mathbb{R}$ ([2]), was inspired by a remark in [1] suggesting a very natural involvement of this notion in studying sweeping processes.

A very wide category of problems is thus covered, such as usual differential problems, impulsive inclusions, dynamic equations on time scales and generalized differential equations since the map g may have discontinuities (leading to jumps in the state) or intervals where it is constant (leading to stationary periods of time for the state).

Here $N_{C(t, u(t))}(u(t))$ denotes the normal cone of $C(t, u(t))$ at the point $u(t)$.

References

- [1] J.J. Moreau, Evolution problem associated with a moving convex set in a Hilbert space, *J. Diff. Equations*, 26(3), 347–374 (1977).
- [2] R. López Pouso, A. Rodríguez, A new unification of continuous, discrete, and impulsive calculus through Stieltjes derivatives. *Real Anal. Exch.*, 40, 319–353 (2015).
- [3] B. Satco, G. Smyrlis, *State-dependent sweeping processes with Stieltjes derivative*, *Applied Math. Optim.* 90:24, 2024.